Bicycle Model and The Understeer Budget

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About the author

Tim Drotar is currently a lead engineer in advanced vehicle dynamics at Stellantis. Previously, he spent 30 years at Ford Motor Company where he specialized in chassis systems and vehicle dynamics for passenger cars and light trucks. Tim is a member of SAE, SCCA and The Tire Society. He holds a B.S. in Mechanical Engineering from Lawrence Technological University and a M.S. in Mechanical Engineering from the University of Michigan-Dearborn.

Tim also teaches the following classes for SAE:

- Advanced Vehicle Dynamics for Passenger Cars and Light Trucks
  - https://www.sae.org/learn/content/c0415/
- Fundamentals of Steering Systems
  - https://www.sae.org/learn/content/c0716/

Introduction

This course **is** an attempt to show how we can use the bicycle model and the understeer budget to:

- Cascade vehicle handling metrics to suspension, steering and tire characteristics
- Assemble suspension, steering and tire characteristics to predict vehicle handling response performance
- Get us thinking of vehicle dynamics and chassis from a ‘Systems Engineering V’ perspective

Introduction

This course <u>**is not**</u> instruction in:

- Deriving or solving the equations of motion for the bicycle model nor the equations for the components of the understeer budget. This has already been done in literature.
- Suspension kinematics and compliances (K&C). This assumes the attendee has some familiarity with the basic tests and metrics.
  - If not, consider looking at the following resources:
    - https://www.morsemeasurements.com/
      - Good video presentations such as “What is K&C Testing?” and “K&C Test Descriptions”
    - SAE seminar C0415 – Advanced Vehicle Dynamics for Passenger Cars and Light Trucks (https://www.sae.org/learn/content/c0415/)
    - SAE papers
      - “A New Laboratory Facility for Measuring Vehicle Parameters Affecting Understeer and Brake Steer”, A.L. Nedley and W. J. Wilson, SAE paper no. 720473, SAE International
      - “Steering and Suspension Test and Analysis”; K. VanGorder, T. David and J. Basas, SAE paper no. 2001-01-1626, SAE International
      - “A Facility for the Measurement of Heavy Truck Chassis and Suspension Kinematics and Compliances”; J. Warfford and N. Frey, SAE paper no. 2004-01-2609, SAE International
      - “Using K&C Measurements for Practical Suspension Tuning and Development”; P. Morse, SAE paper no. 2004-01-3547, SAE International

Introduction

This presentation is based on the following SAE papers

"**The Influence of Vehicle Design Parameters on Characteristic Speed and Understeer**", R. T. Bundorf; General Motors Corp., 670078, SAE International, 1967

- In this paper, Bundorf provides an objective definition of understeer and the characteristic speed as it pertains to response gain. He also introduces the concept of cornering compliances and the understeer budget, although he has yet to give them the names. He gives an equation of understeer in terms of chassis design parameters and tire force and moment characteristics

"**A New Laboratory Facility for Measuring Vehicle Parameters Affecting Understeer and Brake Steer**"; A. L. Nedley and W. J. Wilson; General Motors Corp., 720473, SAE International, 1972

- Nedley and Wilson describe the kinematics and compliance (K&C) data required and equations used to calculate understeer, like Bundorf did in 670078, but with additional effects (lateral force camber compliance, for one). The second part of the paper describes the design and operation of GM's 'new' K&C test machine

SAE papers (continued):

"**The Cornering Compliance Concept for Description of Vehicle Directional Control Properties**"; R.T. Bundorf and R.L. Leffert; General Motors Corp., 760713, SAE International; 1976

- In this paper, Bundorf and Leffert formally introduce the concept of cornering compliances. From the bicycle model, they derive the transfer functions for response gain and provide useful relationships to analyze the effect of front and rear cornering compliances on vehicle steady state and transient responses

Introduction

Learning Objectives

- Use the Systems Engineering V and first principals to **cascade** vehicle handling response **to** tire and suspension kinematic and compliance (K&C) parameters
- Relate cornering compliances to vehicle planar handling response through use of the bicycle model
- Specify the components of the lumped cornering compliance
- Determine the relative contribution of tires and K&C on vehicle understeer gradient
- Calculate the Understeer Budget for an exemplar vehicle
- Use the Systems Engineering V and first principles to **predict** vehicle handling response **from** tire and K&C parameters

Outline

1. Introduction
2. **The Systems Engineering V for Vehicle Dynamics**
3. Understeer and Cornering Compliances
4. The Bicycle Model for Handling
5. The Systems Engineering V for Planar Handling
6. The Understeer Budget
7. Calculating the Understeer Budget
8. Estimating Planar Handling Performance
9. Correlation
10. Summary

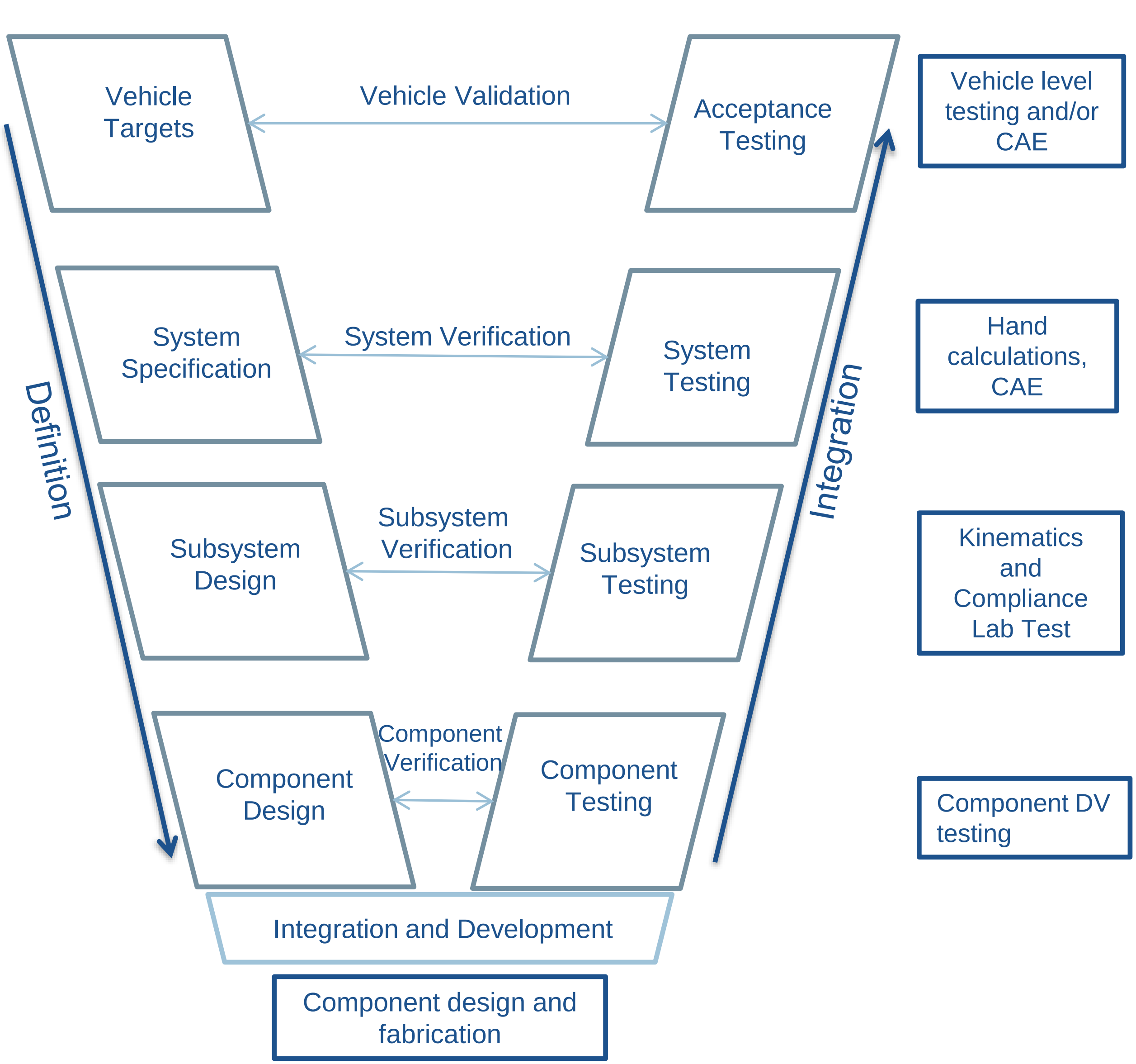
Systems Engineering V for Vehicle Dynamics

Vehicle-level Targets:
- Understeer (deg/g)
- Yaw/SWA time delay (mS)
- Roll gradient (deg/g)
- SS Lat acc gain (g/deg)
- BiT Yaw Rate Overshoot (-)
- Strg torque gradient (Nm/g)
- …

Chassis System Targets:
- Axle roll stiffness (Nm/deg)
- Roll couple distribution (-)
- F/R roll center height (mm)
- F/R cornering compliance (deg/g)
- …

Suspension & Steering Subsystem Targets:
- F/R lateral force compliance steer (deg/kN)
- F/R roll steer (deg/deg)
- F/R stabilizer bar contrib. to roll stiffness (%)
- Caster trail (mm)
- …

Component Specifications
- Suspension hardpoints (xyz)
- Lwr steering hardpoints (xyz)
- Strg gear ratio (mm/rev)
- Suspension bushing stiffness
- Tire force and moments
- Power steering assist tuning
- Shock absorber tuning
- …



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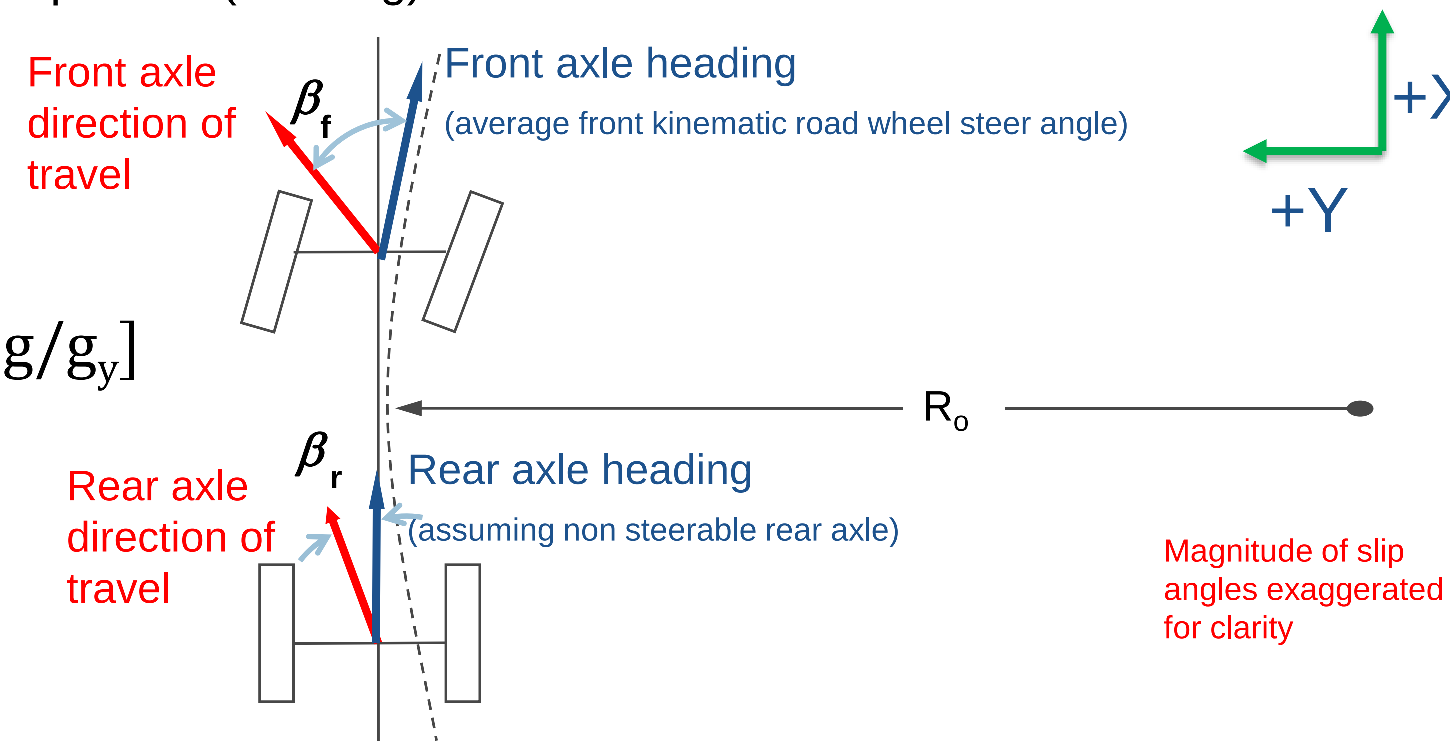
Sideslip Angle and Cornering Compliances

The side slip angle is the angular difference between the direction you are travelling and direction you are pointed (heading).

$$D = \frac{d\beta}{da_y} \quad [\mathrm{deg}/\mathrm{g_y}]$$



Axle Cornering Compliance, *D*, is the rate of change of axle sideslip angle, β, with increasing lateral acceleration.

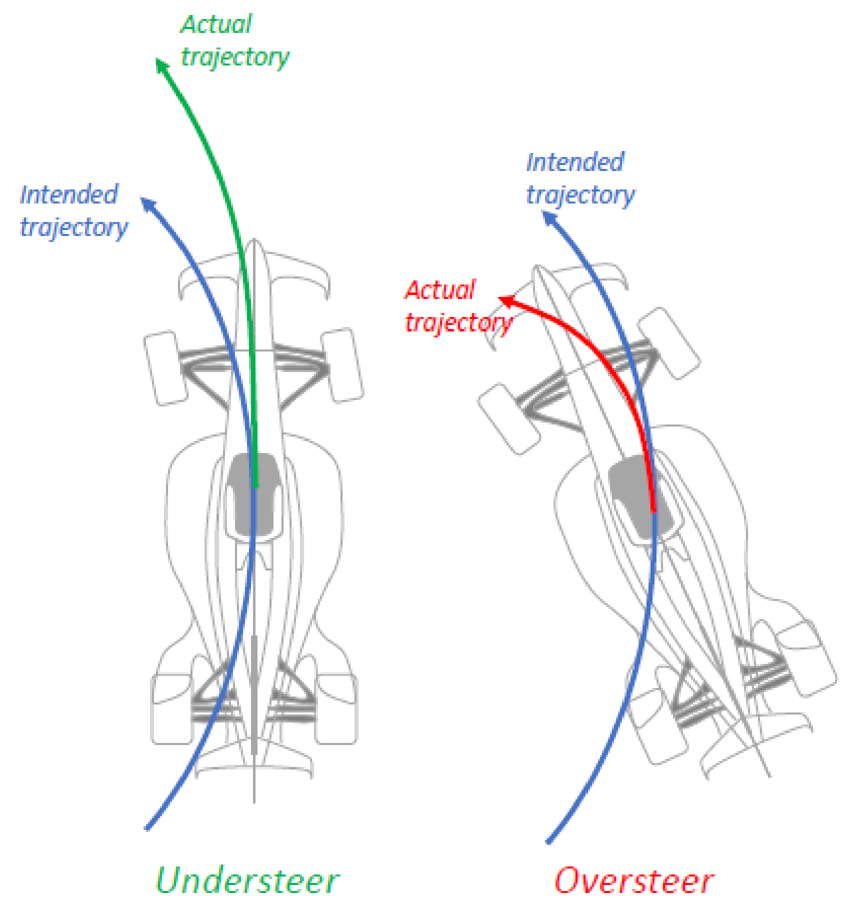
The **Understeer Gradient, K_{us}** is the difference between the front and rear axle cornering compliance.

Understeer, Oversteer and Neutral Steer (a working definition)

Understeer:
The car is rotating (yawing) less that you intended it to.

Oversteer:
The car is rotating (yawing) more than you intended it to.

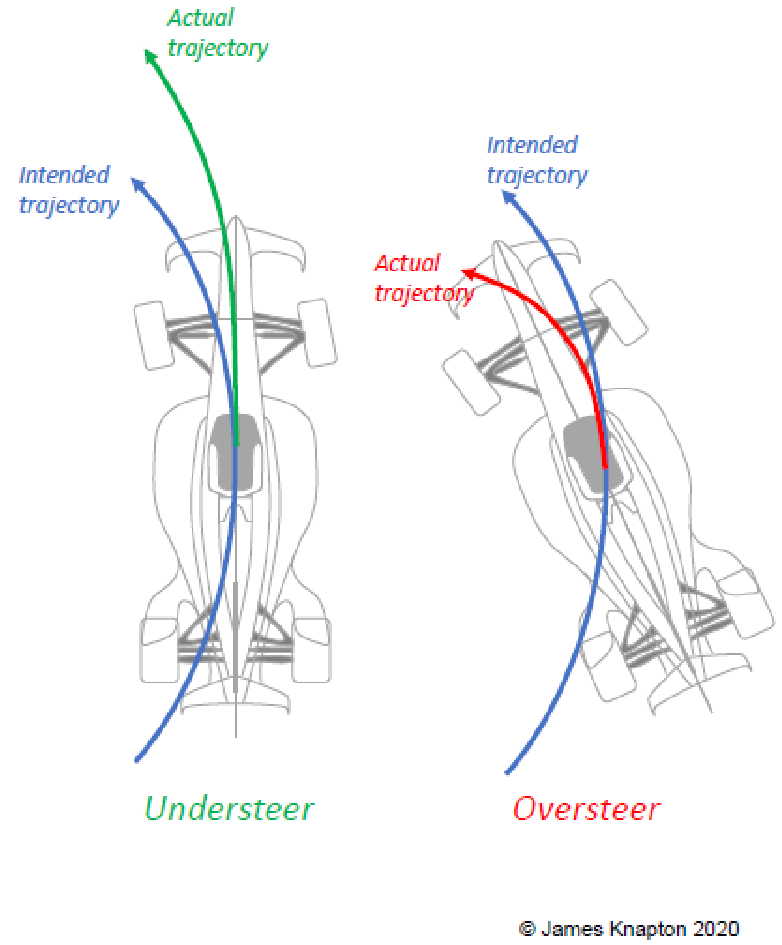
Neutral Steer:
The car is rotating (yawing) as you intended it to.



Understeer, Oversteer and Neutral Steer (an analytical definition)

Understeer Gradient, K_{us},

- Is the difference between the front and rear cornering compliances, D_f and D_r
- Units are deg axle sideslip per g of lateral acceleration



Understeer:

Understeer Gradient (K_{us}) greater than zero

Oversteer:

Understeer Gradient (K_{us}) less than zero

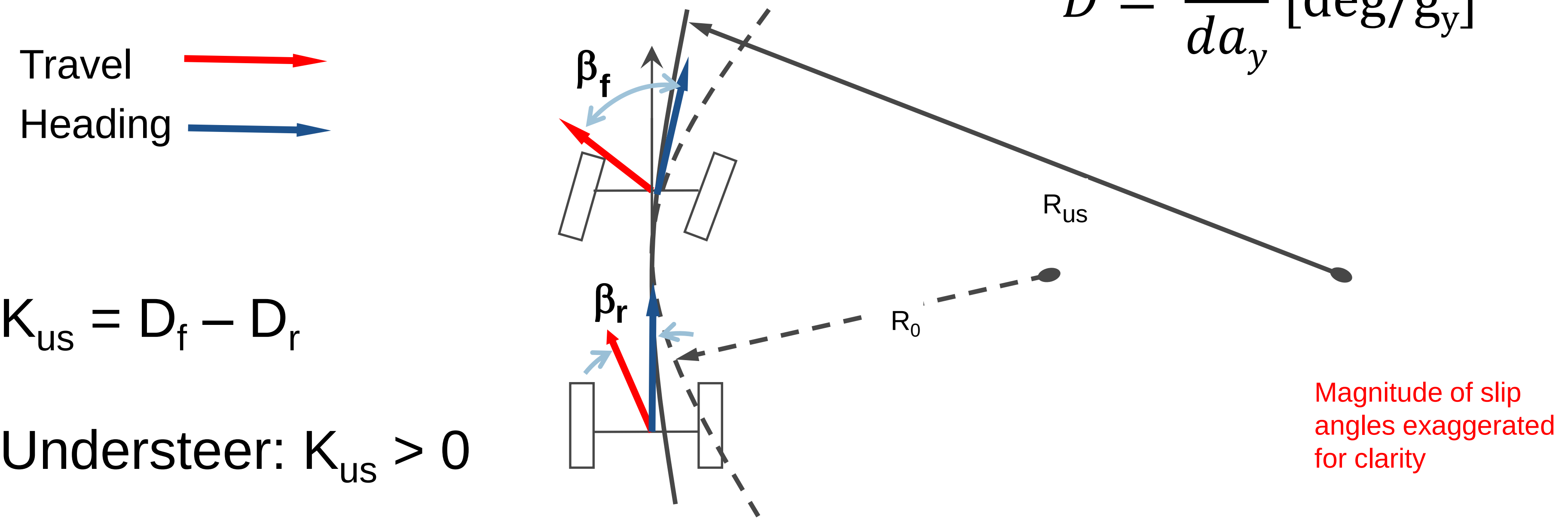
Neutral Steer:

Understeer Gradient (K_{us}) equal to zero

Definition of Understeer in Terms of Cornering Compliances

Understeer occurs when the front axle cornering compliance is greater than the rear axle cornering compliance, i.e. $d\beta_f/da_y > d\beta_r/da_y$

$$D = \frac{d\beta}{da_y}\ [\mathrm{deg/g_y}]$$



$K_{us} = D_f - D_r$

Understeer: $K_{us} > 0$

Driving around a fixed radius circle at ever increasing speed (thus, increasing lateral acceleration), while holding a constant steering wheel angle, the vehicle would follow a larger radius path. Otherwise, the driver would be required to increase steering wheel angle to stay on path.

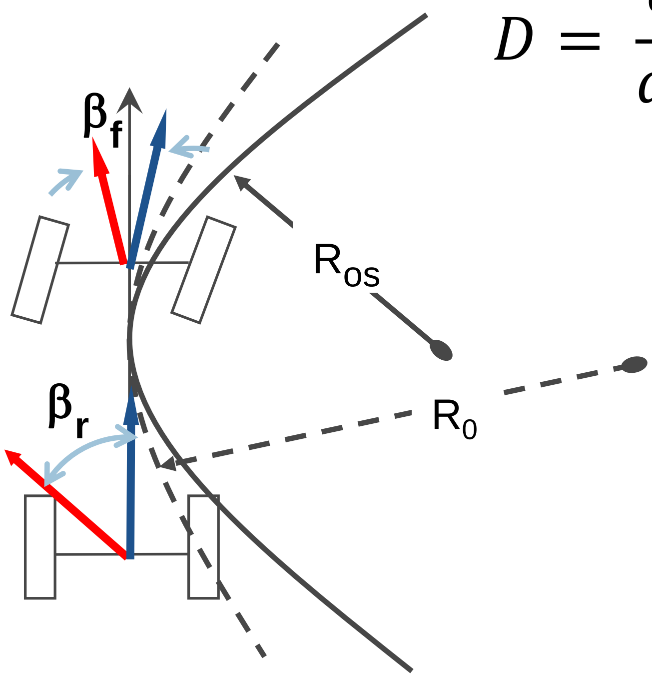
Definition of Oversteer in Terms of Cornering Compliances

Oversteer occurs when the front axle cornering compliance is less than the rear axle cornering compliance, i.e. $d\beta_f / da_y < d\beta_r / da_y$

$$D = \frac{d\beta}{da_y}\ [\mathrm{deg/g_y}]$$

Travel

Heading



$K_{us} = D_f - D_r$

Oversteer: $K_{us} < 0$

Magnitude of slip angles exaggerated for clarity

Driving around a fixed radius circle at ever increasing speed (thus, increasing lateral acceleration), while holding a constant steering wheel angle, the vehicle would follow a smaller radius path. Otherwise, the driver would be required to decrease steering wheel angle to stay on path.

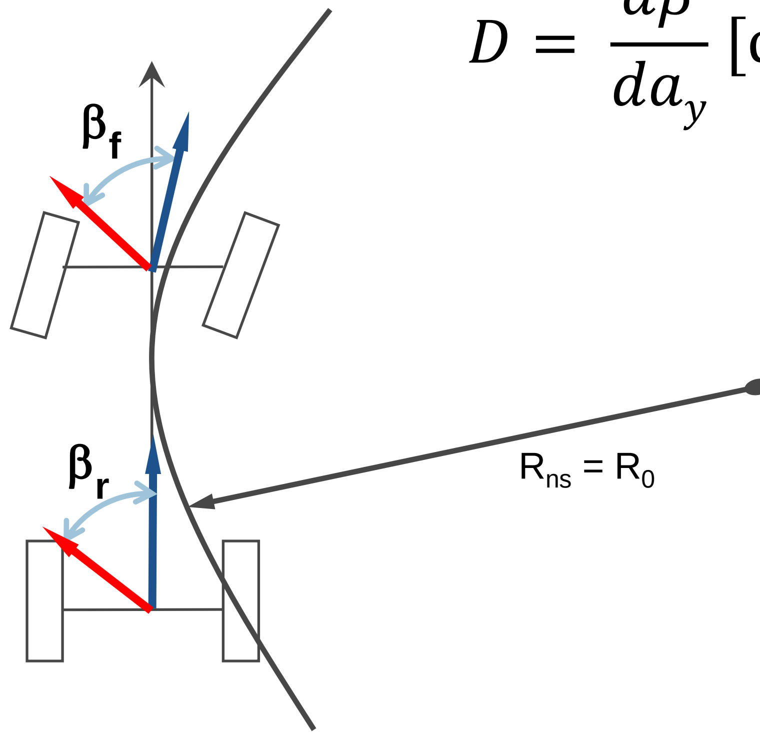
Definition of Neutral Steer in Terms of Cornering Compliances

Neutral steer occurs when the front axle cornering compliance is equal to the rear axle cornering compliance, i.e. $d\beta_f / da_y = d\beta_r / da_y$

$$D = \frac{d\beta}{da_y} \; [\mathrm{deg/g_y}]$$

Travel

Heading



$K_{us} = D_f - D_r$

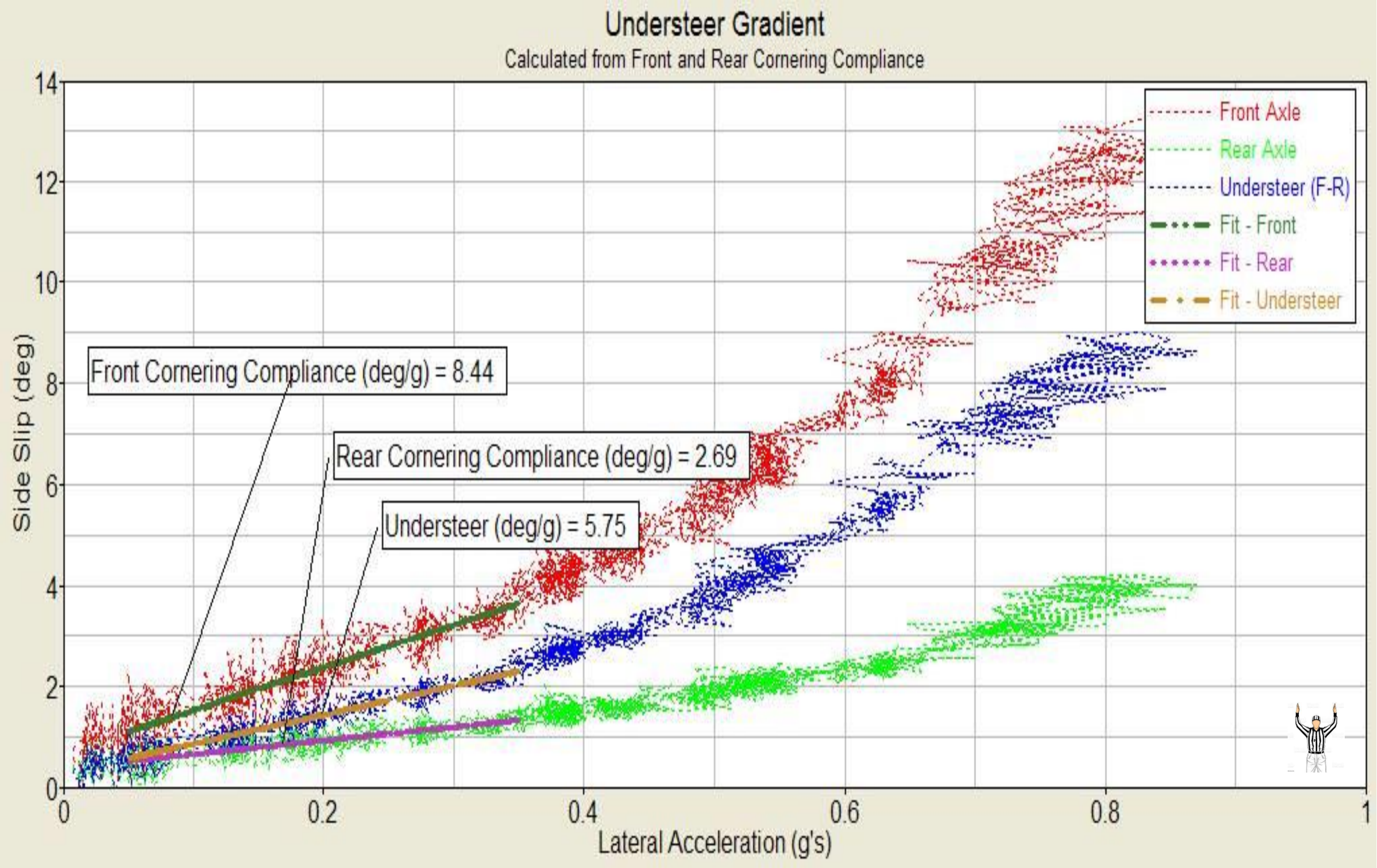
Neutral Steer: $K_{us} = 0$

Magnitude of slip angles exaggerated for clarity

Driving around a fixed radius circle at ever increasing speed (thus, increasing lateral acceleration) while holding a constant steering wheel angle, the vehicle would follow the fixed radius path

Example

Cornering Compliance and Understeer from Constant Speed Understeer Test (aka Swept or Ramp Steer Test)



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The Bicycle Model

The concept of the bicycle model for handling has been around for decades. It is well documented in literature. Hence, we will not solve the equations of motion here.

Google Scholar search for "Bicycle Model Vehicle Dynamics" returned 104k hits

A few good references on the bicycle model:

"The Cornering Compliance Concept for Description of Vehicle Directional Control Properties"; R.T. Bundorf and R.L. Leffert, SAE #760713, SAE International.

"Vehicle Dynamics"; J. R. Ellis, London Business Books, 1969

"The Complex Cornering Compliance Theory and its Application to Vehicle Dynamics Characteristics"; K. Tsuji and N. Totoki, SAE #2002-01-1218, SAE International

We will use Bundorf and Leffert's (B&L) derivation and solution in the following discussions

The B&L Bicycle Model – A Summary

$M_v * u * (\dot{\beta}_v + r) = \Sigma F_y$

$J_{v,z} * \dot{r} = \Sigma M_z$

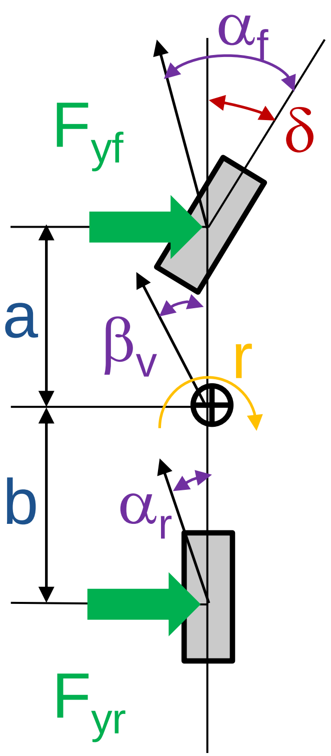
(1)

$$\begin{bmatrix} M_v u \boldsymbol{s} + C_f + C_r & M_v u + \frac{aC_f}{u} - \frac{bC_r}{u} \\ aC_f - bC_r & J_{z,v}\boldsymbol{s} + \frac{a^2 C_f}{u} - \frac{b^2 C_r}{u} \end{bmatrix} \begin{bmatrix} \bar{\beta}_v \\ \bar{r} \end{bmatrix} = \begin{bmatrix} C_f \\ aC_f \end{bmatrix} * \delta$$

Derivative notation to Laplace notation

$\dot{\beta}_v = \bar{\beta}_v s$

$\dot{r} = \bar{r} s$



M_v	= vehicle mass, kg
$J_{z,v}$	= vehicle inertia, kg-m^2
u	= forward velocity, m/s
a, b	= distance from cg to front, rear axle, m
C_f, C_r	= front and rear cornering stiffness, N/radian
α_f, α_r	= front and rear slip angles, radians
δ	= front steer angle, radians
F_{yf}, F_{yr}	= front and rear lateral forces, N
r	= yaw velocity, radians/sec
β_v	= vehicle sideslip angle, radians
s	***= Laplace operator***

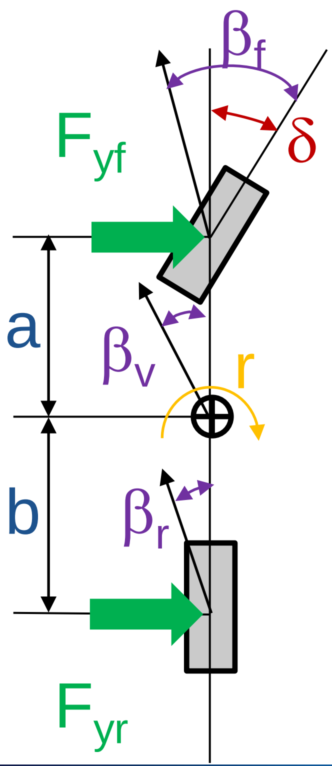
(1) SAE paper 760713 Appendix B: Directional Control Equations for a Simple Non-Rolling Vehicle Model

Converting cornering stiffness to cornering compliance:

$$D_f\left(\frac{\text{deg}}{g}\right) = 57.3 * \frac{M_v g b}{(a+b)} * \frac{1}{C_f} \qquad D_r\left(\frac{\text{deg}}{g}\right) = 57.3 * \frac{M_v g a}{(a+b)} * \frac{1}{C_r}$$

<u>Note Conversion</u>:
57.3 degrees = 1 radian

The state space equations become:



$$\begin{bmatrix} M_v u \boldsymbol{s} + \frac{57.3 M_v g}{(a+b)}\left[\frac{aD_f + bD_r}{D_f D_r}\right] & M_v u - \frac{57.3 M_v ab}{u(a+b)}\left[\frac{D_f - D_r}{D_f D_r}\right] \\ -\frac{57.3 M_v gab}{(a+b)}\left[\frac{D_f - D_r}{D_f D_r}\right] & J_{z,v}\boldsymbol{s} - \frac{57.3 M_v gab}{u(a+b)}\left[\frac{bD_f - aD_r}{D_f D_r}\right] \end{bmatrix} \begin{bmatrix} \bar{\beta}_v \\ \bar{r} \end{bmatrix} = \begin{bmatrix} \frac{57.3 M_v g b}{(a+b) D_f} \\ \frac{57.3 M_v gab}{(a+b) D_f} \end{bmatrix} * \ \delta$$

M_v	= vehicle mass, kg
$J_{z,v}$	= vehicle inertia, kg-m^2
v_f	= forward velocity, m/s
a, b	= distance from cg to front, rear axle, m
g	= acceleration due to gravity = 9.806m/s^2
C_f, C_r	= front and rear cornering stiffness, N/radian
D_f, D_r	**= front and rear cornering compliance, deg/g**
β_f, β_r	= front and rear axle sideslip slip angles, radians
δ	= front steer angle, radians
F_{yf}, F_{yr}	= front and rear lateral forces, N
r	= yaw velocity, radians/sec
β_v	= vehicle sideslip angle, radians
s	***= Laplace operator***

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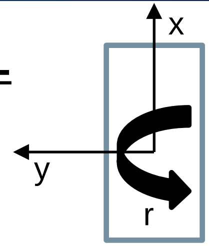
The B&L Bicycle Model Transfer Function – Yaw Rate

Vehicle Yaw Rate / Steering Wheel Angle Gain from B&L

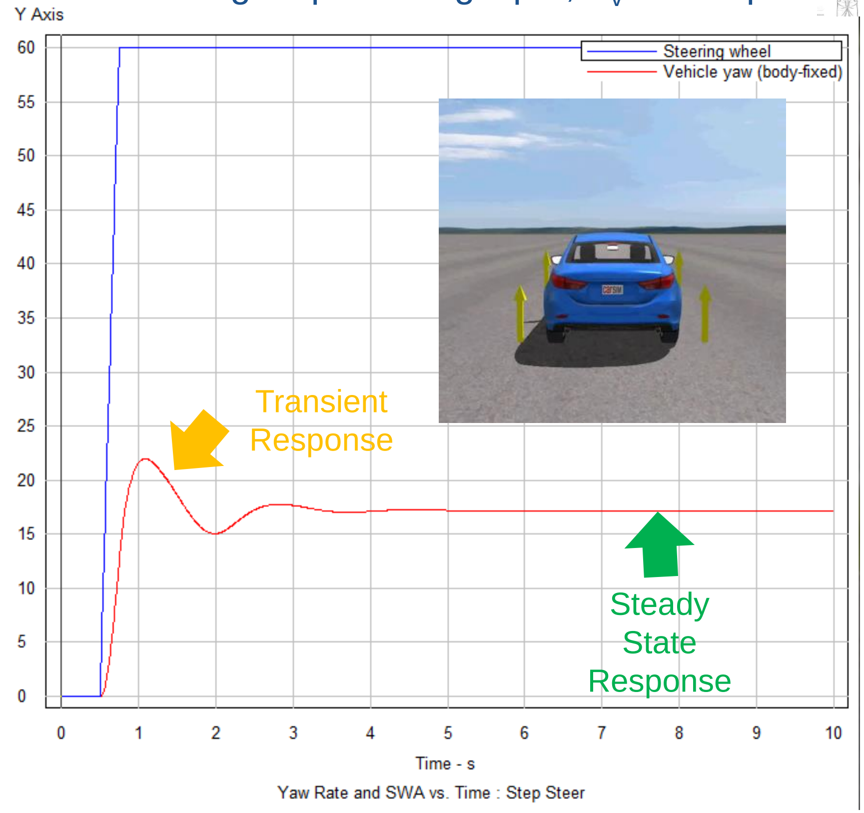


$$\frac{r}{\delta_{sw}} = \frac{1}{i} * \frac{\left[\frac{D_r u^2}{57.3g}\right]s+1}{\left[\frac{\frac{(\frac{k_{z,v}^2}{ab})u^2 D_f D_r}{(57.3g)^2}}{1+\frac{u^2(D_f-D_r)}{57.3g(a+b)}}\right]s^2 + \left[\frac{\frac{u(D_f-D_r)}{57.3g}+\frac{u(\frac{k_{z,v}^2}{ab}-1)(aD_f+bD_r)}{57.3g(a+b)}}{1+\frac{u^2(D_f-D_r)}{57.3g(a+b)}}\right]s+1} * \left(\frac{r}{\delta_{sw}}\right)_{ss}$$

$$\left(\frac{r}{\delta_{sw}}\right)_{ss} = \frac{1}{i} * \frac{\frac{u}{a+b}}{1+\frac{u^2(D_f-D_r)}{57.3g(a+b)}}$$

Steady State
Yaw Gain

Exemplar vehicle yaw response
60 deg step steering input, V_v = 100kph



Notice that the cornering compliances, D_f and D_r appear throughout the transfer function, individually, as sums, differences and products.

Recall

- The Understeer Gradient: $K_{us} = D_f - D_r$
- $k^2_{z,v} = J_{z,v} / M_v$
- Overall steering ratio: i [deg swa/1deg rwa]

Vehicle Yaw Rate Response Time from B&L

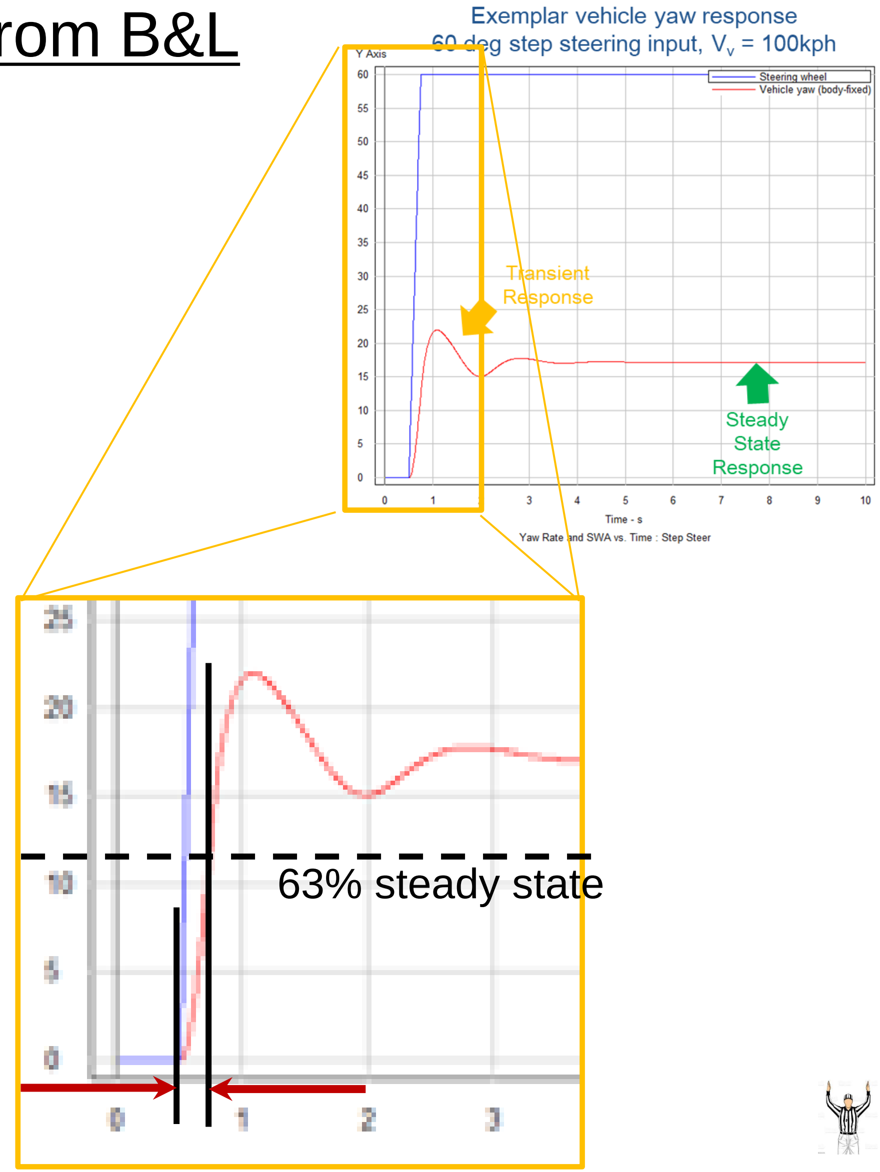
$$Yaw\ Rate\ Response\ Time \cong \frac{D_f u \frac{k^2}{ab} \frac{1}{57.3g}}{1 + \frac{u^2 (D_f - D_r)}{57.3g(a+b)}}$$

According to B&L:
The values of response time acquired from this expression will most closely correspond to those referenced to a 'time to 63% steady state' response during a step steer maneuver

Notice that changes to the front and rear cornering compliances, D_f and D_r , affect the response time

Recall
- The Understeer Gradient: $K_{us} = D_f - D_r$
- $k^2_{z,v} = J_{z,v} / M_v$

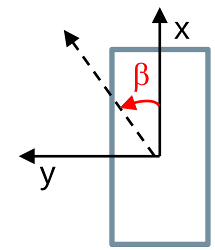
$Yaw\ Rate\ Response\ Time\ (s)$



Vehicle Side Slip Angle / Steering Wheel Angle Gain from B&L

$$\frac{\beta_{vm}}{\delta_{sw}} = \frac{1}{i} * \frac{\dfrac{\dfrac{D_r u}{57.3g}\left(1 + \dfrac{2k_{z,v}^2}{a(a+b)}\right)}{1 - \dfrac{2D_r u^2}{57.3g(a+\ b)}}}{\left[\dfrac{\dfrac{(\frac{k_{z,v}^2}{ab})u^2 D_f D_r}{(57.3g)^2}}{1 + \dfrac{u^2(D_f - D_r)}{57.3g(a+b)}}\right]s^2 + \left[\dfrac{\dfrac{u(D_f - D_r)}{57.3g} + \dfrac{u(\frac{k_{z,v}^2}{ab} - 1)(aD_f + bD_r)}{57.3g(a+b)}}{1 + \dfrac{u^2(D_f - D_r)}{57.3g(a+b)}}\right]s + 1} * \left(\frac{\beta_{vm}}{\delta_{sw}}\right)_{ss}$$

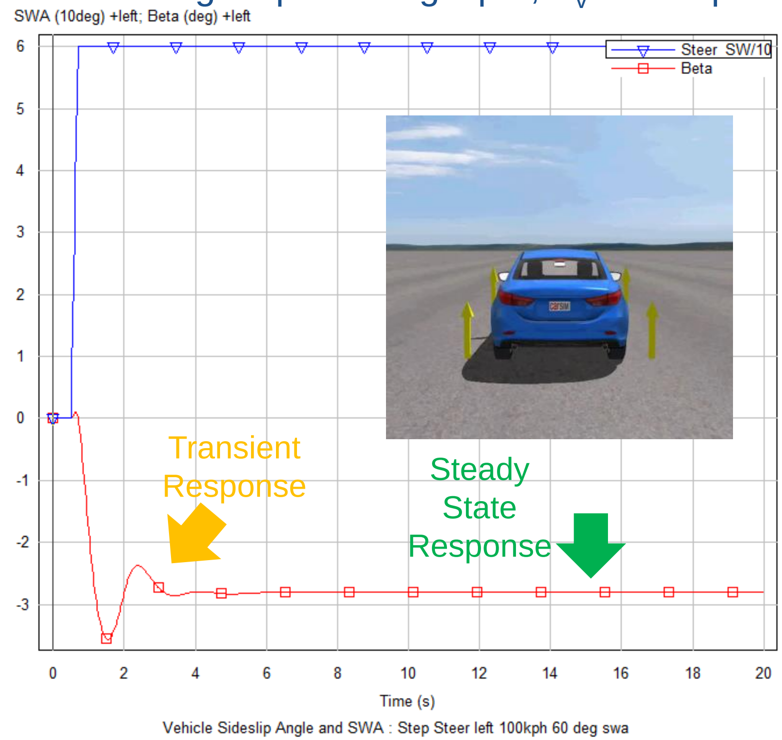
Magnitude of side slip angle exaggerated for clarity



$$\left(\frac{\beta_{vm}}{\delta_{sw}}\right)_{ss} = \frac{1}{i} * \frac{\dfrac{1}{2} - \dfrac{D_r u^2}{57.3g(a+b)}}{1 + \dfrac{u^2(D_f - D_r)}{57.3g(a+b)}}$$

Steady State Sideslip Angle Gain
(mid-wheelbase reference point)

Exemplar vehicle side slip angle response
60 deg step steering input, V_v = 100kph



Notice that the cornering compliances, D_f and D_r appear throughout the transfer function, individually, as sums, differences and products.

Recall

- The Understeer Gradient: $K_{us} = D_f - D_r$
- $k^2{}_{z,v} = J_{z,v} / M_v$
- Overall steering ratio: i [deg swa/1deg rwa]

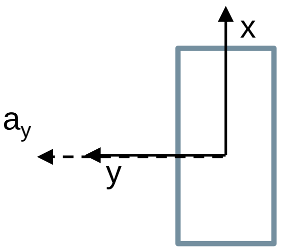
The B&L Bicycle Model Transfer Function – Lat Acc

Lateral Acceleration / Steering Wheel Angle Gain from B&L

Although we don’t get the lateral acceleration explicitly from the state space equations, we can calculate the response at the vehicle midpoint using the following relationship:

$$\frac{a_{ym}}{\delta}(s) = u[s\frac{\beta_{vm}}{\delta}(s) + \frac{r}{\delta}(s)]$$

Lateral Acceleration / Steering Wheel Angle Gain from B&L



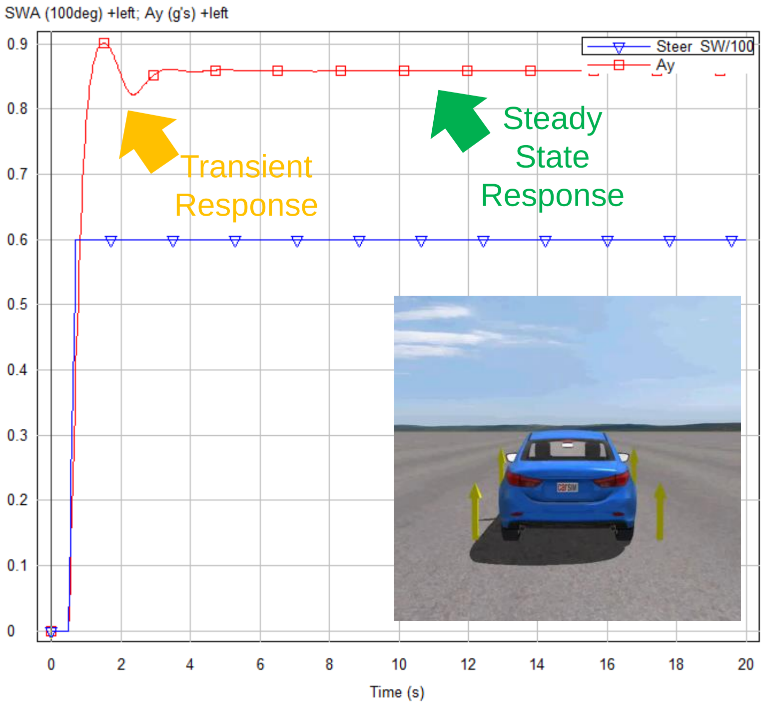
$$\frac{a_{ym}}{\delta_{sw}} = \frac{1}{i} * \frac{\left[\frac{D_r}{57.3g}\left(\frac{k_{z,v}^2}{a}+\frac{a+b}{2}\right)\right]s^2 + \left[\frac{a+b}{2u}\right]s+1}{\left[\frac{\frac{(\frac{k_{z,v}^2}{ab})u^2 D_f D_r}{(57.3g)^2}}{1+\frac{u^2(D_f-D_r)}{57.3g(a+b)}}\right]s^2 + \left[\frac{\frac{u(D_f-D_r)}{57.3g}+\frac{u(\frac{k_{z,v}^2}{ab}-1)(aD_f+bD_r)}{57.3g(a+b)}}{1+\frac{u^2(D_f-D_r)}{57.3g(a+b)}}\right]s+1} * \left(\frac{a_{ym}}{\delta_{sw}}\right)_{ss}$$

$$\left(\frac{a_{ym}}{\delta_{sw}}\right)_{ss} = \frac{1}{i} * \frac{\frac{u^2}{a+b}}{1+\frac{u^2(D_f-D_r)}{57.3g(a+b)}}$$

Steady State Lat Acc Gain

(mid-wheelbase reference point)

Exemplar vehicle lat acc response
60 deg step steering input, V_v = 100kph



Lat Acc and SWA : Step Steer left 100kph 60 deg swa

<u>Notice</u> that the cornering compliances, D_f and D_r appear throughout the transfer function, individually, as sums, differences and products.

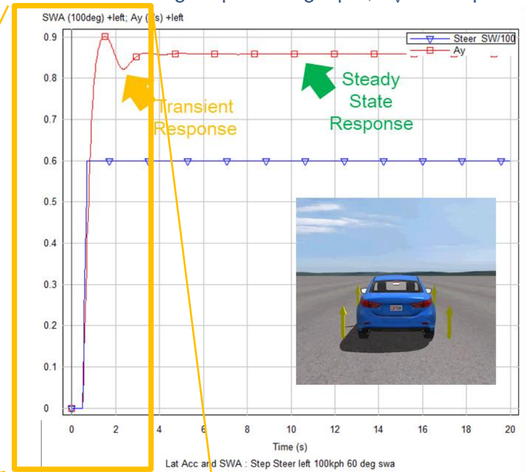
<u>Recall</u>

- The Understeer Gradient: $K_{us} = D_f - D_r$
- $k^2_{z,v} = J_{z,v} / M_v$
- Overall steering ratio: i [deg swa/1deg rwa]

The B&L Bicycle Model – Lat Acc Response Time

Approximate Time to Peak Lateral Acceleration from B&L

Exemplar vehicle lat acc response
60 deg step steering input, V_v = 100kph



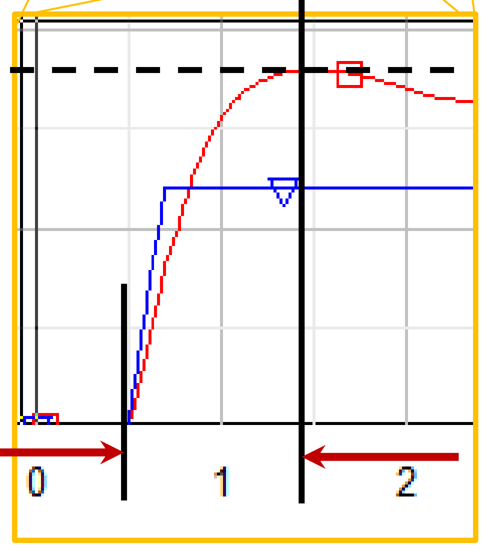
$$\mathit{Approximate\ Time\ to\ Peak\ Lateral\ Acceleration} \cong 2\sqrt{\frac{D_f D_r (a+b)\dfrac{k_{z,v}^2}{ab}}{(D_f - D_r)57.3g}}$$

Notice that changes to the front and rear cornering compliances, D_f and D_r , affect the response time

Recall

- The Understeer Gradient: $K_{us} = D_f - D_r$
- $k^2_{z,v} = J_{z,v} / M_v$

Time to Peak Lat Acc (s)



What we learned so far:

- Axle sideslip angle is the angular difference between the direction the axle is travelling and direction it is pointed

- Cornering compliance is the change in axle sideslip angle per g of lateral acceleration. By sign convention (either SAE or ISO)
  - Positive Cornering Compliance at the front axle is **UNDERSTEER**
  - Positive Cornering Compliance at the rear axle is **OVERSTEER**

- The Bicycle Model can be used to estimate the steady state and transient response of a vehicle to a step steering input

- The magnitude of the response is a function of
  - Vehicle mass and inertia
  - Vehicle speed
  - Overall steering ratio
  - Front and rear cornering compliances

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5. **The Systems Engineering V for Planar Handling**
6. The Understeer Budget
7. Calculating the Understeer Budget
8. Estimating Planar Handling Performance
9. Correlation
10. Summary

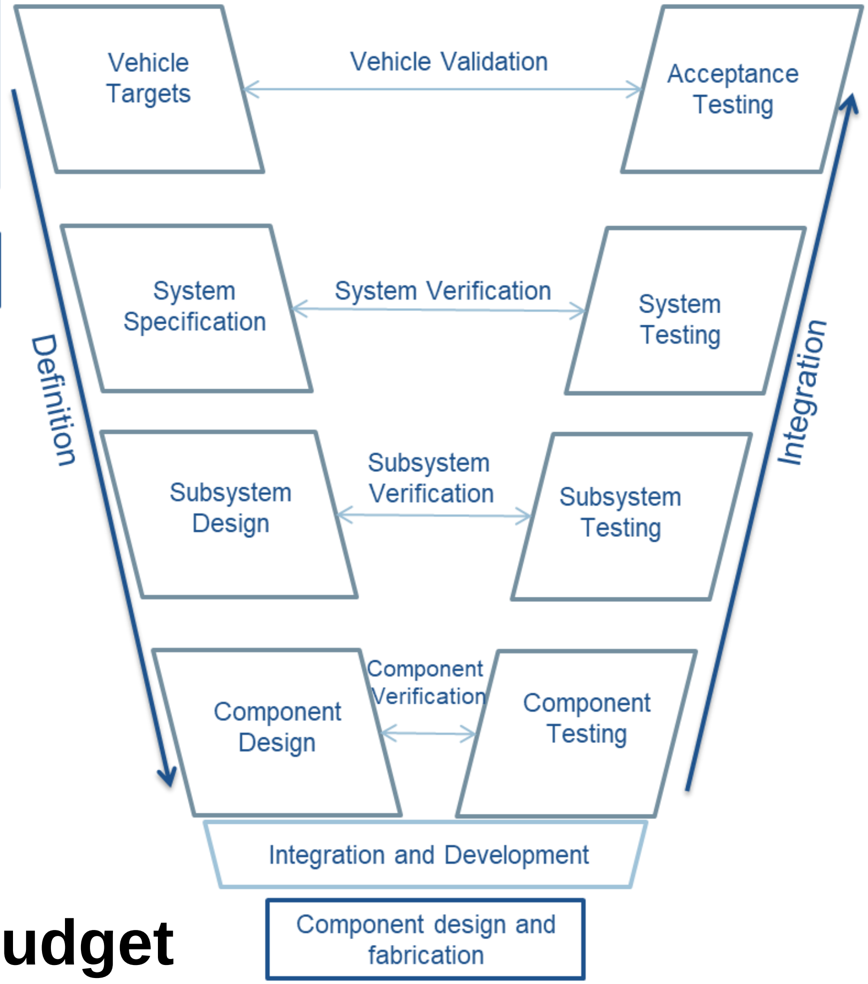
Q: How do we relate front and rear cornering compliances to suspension kinematics/compliances and tire properties?

Vehicle-level Targets:
- Understeer Gradient
- Yaw Rate / SWA gain
- Yaw Rate/SWA response time
- Ay / SWA gain
- Ay / SWA response time
- Sideslip Angle / SWA gain

Chassis System Targets:
- F/R cornering compliance (deg/g)

Suspension & Steering Subsystem Targets:
- F/R lateral force compliance steer (deg/kN)
- F/R aligning torque compliance steer (deg/kN-m)
- F/R camber compliance (deg/kN)
- F/R roll steer (deg/deg)
- F/R roll camber (deg/deg)

Component Specifications
- Suspension hardpoints (xyz)
- Lwr steering hardpoints (xyz)
- Strg gear ratio (m/rev)
- Suspension bushing stiffness
- Steering system stiffness
- Tire force and moments



Vehicle level testing and/or CAE

Hand calculations, CAE

Kinematics and Compliance Lab Test

Component DV testing

A: **The Understeer Budget**

Outline

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The Understeer Budget

The front and rear axle cornering compliances are actually '**lumped compliances**', comprised of the cornering compliance due to tire cornering and aligning torque stiffness, suspension kinematics and compliances.

We can convert K&C parameters from their native units to cornering compliance (deg sideslip / g of lat acc) and sum to get total axle cornering compliance.

Front Axle:
Front Weight & Tire Effect
\+ Front Suspension Kinematic Effect
\+ Front Suspension Compliance Effect
= Front Cornering Compliance (D_f)

Rear Axle:
Rear Weight & Tire Effect
\+ Rear Suspension Kinematic Effect
\+ Rear Suspension Compliance Effect
= Rear Cornering Compliance (D_r)

$$\Rightarrow \quad K_{us} = D_f - D_r$$

This is what we refer to as "**The Understeer Budget**"

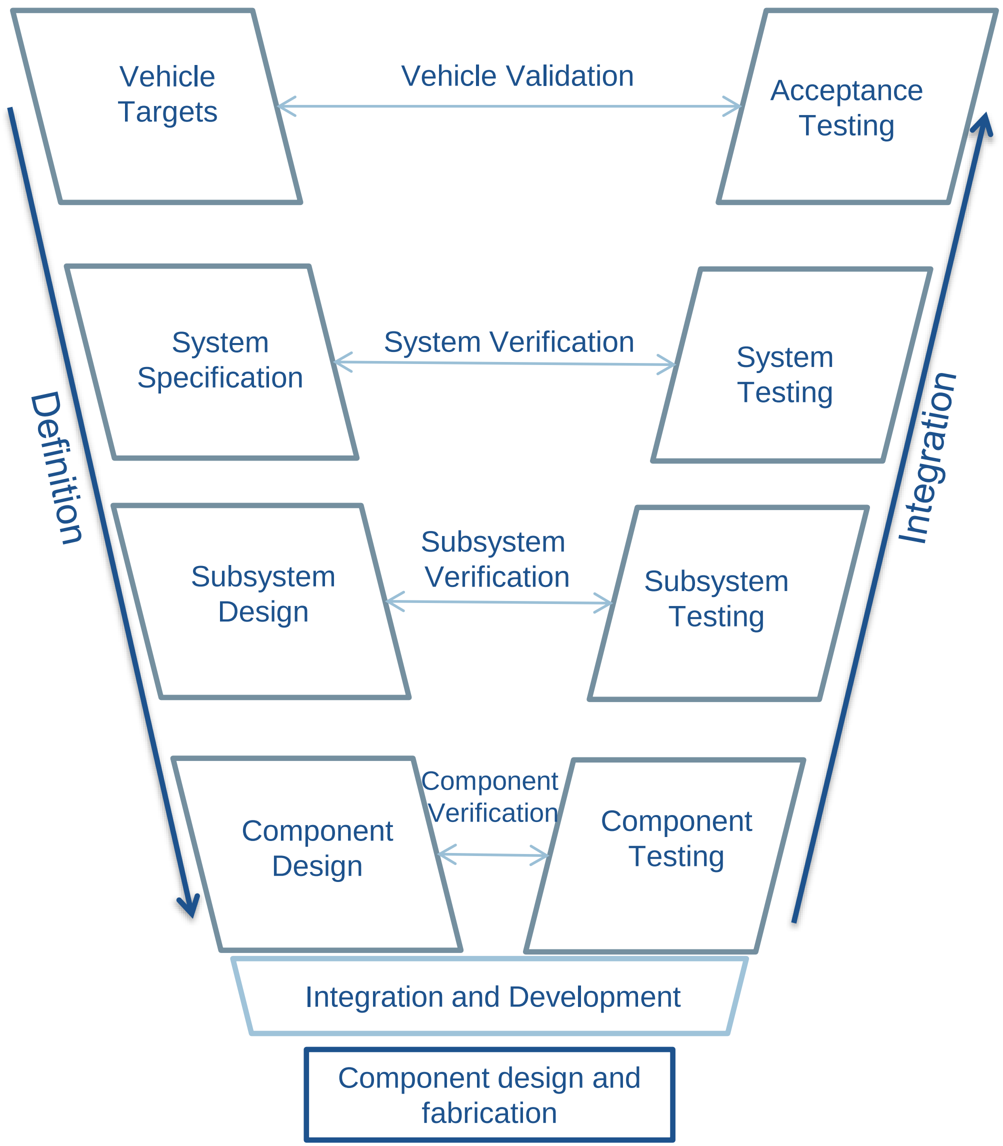
Systems Engineering V – The Understeer Budget

Vehicle-level Targets:
- Understeer Gradient
- Yaw Rate / SWA gain
- Yaw Rate/SWA response time
- Ay / SWA gain
- Ay / SWA response time
- Sideslip Angle / SWA gain

Chassis System Targets:
- F/R cornering compliance (deg/g)
  - F/R lateral force compliance steer
  - F/R aligning torque compliance steer
  - F/R roll steer
  - F/R roll camber
  - Tire cornering compliance

Suspension & Steering Subsystem Targets:
- F/R lateral force compliance steer (deg/kN)
- F/R aligning torque compliance steer (deg/kN-m)
- F/R camber compliance (deg/kN)
- F/R roll steer (deg/deg)
- F/R roll camber (deg/deg)

Component Specifications
- Suspension hardpoints (xyz)
- Lwr steering hardpoints (xyz)
- Strg gear ratio (m/rev)
- Suspension bushing stiffness
- Steering system stiffness
- Tire force and moments

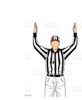


Vehicle level testing and/or CAE

Hand calculations, CAE

Kinematics and Compliance Lab Test

Component DV testing



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In Bundorf's paper "The Influence of Vehicle Design Parameters on Characteristic Speed and Understeer", he presented a general equation for characteristic speed

R. T. BUNDORF

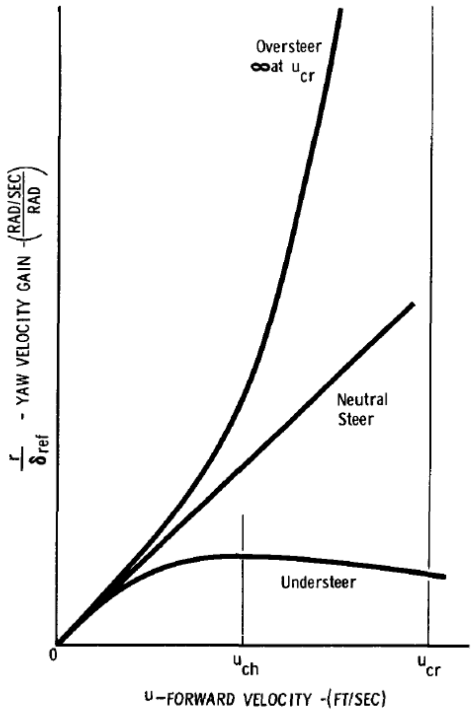


Fig. 1 - Yaw velocity gain versus speed

$$u_{ch} = \sqrt{\frac{57.3\,(a+b)\,g}{K}} \qquad (6)$$

u_{ch} = Characteristic speed (ft/s)

g = 32.2 ft/s^2

a = Distance from vehicle Cg to front axle (ft)

b = Distance from vehicle Cg to rear axle (ft)

K = Understeer Gradient (deg/g)

The **characteristic speed** is defined as the speed at which the straight-ahead control sensitivity of an understeering vehicle is one-half that of a neutral steer vehicle

He then expanded the understeer gradient (denominator) to be a function of tire parameters, suspension kinematics and suspension compliances

$$u_{ch}^2 = \frac{g\,(a+b)\,(57.3)}{\left\{ \frac{W_f}{2C_{\alpha f}}\left(1 + E_{af} N_{\alpha f}\right)\left(1 + \frac{N_{\alpha r}}{bC_{\alpha r}}\right) - \frac{W_r}{2C_{\alpha r}}\left(1 - E_{ar} N_{\alpha r}\right)\left(1 - \frac{N_{\alpha f}}{aC_{\alpha f}}\right) + E_{ff}\frac{W_{sf}}{2} + E_{fr}\frac{W_{sr}}{2} + K'_{\varphi}\left[\left(1 + E_{af} N_{\alpha f}\right)\frac{C_{\gamma f}\Gamma_f}{C_{\alpha f}} + \left(1 - E_{ar} N_{\alpha r}\right)\frac{C_{\gamma r}(-\Gamma_r)}{C_{\alpha r}} + E_{sf} + E_{sr}\right]\right\}} \qquad (7)$$

K (deg/g)

Where:

W_f, W_r = Frt, Rr vehicle weight (N)
W_{sf}, W_{sr} = Frt, Rr sprung weight (N)
$C_{\alpha f}$, $C_{\alpha r}$ = Frt, Rr tire cornering stiffness (N/deg)
$N_{\alpha f}$, $N_{\alpha r}$ = Frt, Rr tire aligning torque stiffness (N-m/deg)
$C_{\gamma f}$, $C_{\gamma r}$ = Frt, Rr tire camber stiffness (N/deg)

Γ_f, Γ_r = Frt, Rr roll inclination angle coef. (deg/deg)
K'_{φ} = Vehicle roll gradient (deg/g_y)
E_{af}, E_{ar} = Frt, Rr aligning torque compliance steer (deg/Nm)
E_{ff}, E_{fr} = Frt, Rr lateral force compliance steer (deg/N)
E_{sf}, E_{sr} = Frt, Rr roll steer coef. (deg/deg)

A few observations:

1/ Notice the interaction between tire, kinematic and compliance parameters

$$u_{ch}^2 = \frac{g\,(a+b)\,(57.3)}{\left\{ \frac{W_f}{2C_{\alpha f}}(1 + E_{af}N_{\alpha f})\left(1 + \frac{N_{\alpha r}}{bC_{\alpha r}}\right) - \frac{W_r}{2C_{\alpha r}}(1 - E_{ar}N_{\alpha r})\left(1 - \frac{N_{\alpha f}}{aC_{\alpha f}}\right) + E_{ff}\frac{W_{sf}}{2} + E_{fr}\frac{W_{sr}}{2} + K'_{\varphi}\left[(1 + E_{af}N_{\alpha f})\frac{C_{\gamma f}\Gamma_f}{C_{\alpha f}} + (1 - E_{ar}N_{\alpha r})\frac{C_{\gamma r}(-\Gamma_r)}{C_{\alpha r}} + E_{sf} + E_{sr}\right]\right\}} \quad (7)$$

K (deg/g)

Where:

W_f, W_r = Frt, Rr vehicle weight (N)

W_{sf}, W_{sr} = Frt, Rr sprung weight (N)

$C_{\alpha f}$, $C_{\alpha r}$ = Frt, Rr tire cornering stiffness (N/deg)

$N_{\alpha f}$, $N_{\alpha r}$ = Frt, Rr tire aligning torque stiffness (Nm/deg)

$C_{\gamma f}$, $C_{\gamma r}$ = Frt, Rr tire camber stiffness (N/deg)

Γ_f, Γ_r = Frt, Rr roll inclination angle coef. (deg/deg)

K'_{φ} = Vehicle roll gradient (deg/g_y)

E_{af}, E_{ar} = Frt, Rr aligning torque compliance steer (deg/Nm)

E_{ff}, E_{fr} = Frt, Rr lateral force compliance steer (deg/N)

E_{sf}, E_{sr} = Frt, Rr roll steer coef. (deg/deg)

2/ The terms $(1 + N_{\alpha r}/bC_{\alpha r)})$ and $(1 - N_{\alpha f}/aC_{\alpha f)})$ capture what Bundorf refers to as ‘the aligning torque effect on the whole vehicle’

$$u_{ch}^2 = \frac{g\,(a + b)\,(57.3)}{\left\{ \frac{W_f}{2C_{\alpha f}}(1 + E_{af}N_{\alpha f})\underbrace{\left(1 + \frac{N_{\alpha r}}{bC_{\alpha r}}\right)} - \frac{W_r}{2C_{\alpha r}}(1 - E_{ar}N_{\alpha r})\underbrace{\left(1 - \frac{N_{\alpha f}}{aC_{\alpha f}}\right)} + E_{ff}\frac{W_{sf}}{2} + E_{fr}\frac{W_{sr}}{2} + K'_{\varphi}\left[(1 + E_{af}N_{\alpha f})\frac{C_{\gamma f}\Gamma_f}{C_{\alpha f}} + (1 - E_{ar}N_{\alpha r})\frac{C_{\gamma r}(-\Gamma_r)}{C_{\alpha r}} + E_{sf} + E_{sr}\right]\right\}} \quad (7)$$

- The ratio of tire aligning torque stiffness, N_α [Nm/deg] to tire cornering stiffness, C_α [N/deg] is know as the **tire pneumatic trail [m]**
- These terms can then be expressed as (b + pneumatic trail)/b and (a - pneumatic trail)/a
- These terms have the effect of shifting the center of gravity forward i.e. increasing W_f and decreasing W_r , thus adding understeer
- Typically, a small adjustment since a and b are on the order of meters, and pneumatic trail is on the order of tens of millimeters

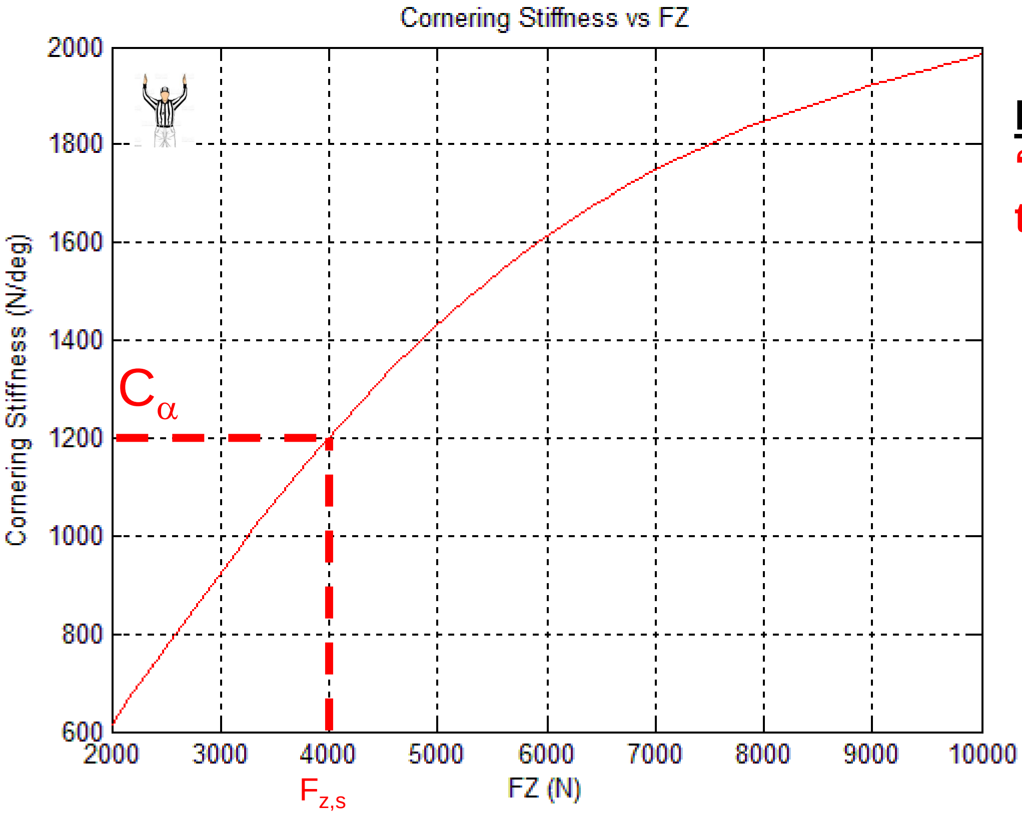
3/ Where are the lateral load transfer terms?

$$u_{ch}^2 = \frac{g\,(a+b)\,(57.3)}{\left\{ \underbrace{\frac{W_f}{2C_{\alpha f}}}_{}(1+E_{af}N_{\alpha f})\left(1+\frac{N_{\alpha r}}{bC_{\alpha r}}\right) - \underbrace{\frac{W_r}{2C_{\alpha r}}}_{}(1-E_{ar}N_{\alpha r})\left(1-\frac{N_{\alpha f}}{aC_{\alpha f}}\right) + E_{ff}\frac{W_{sf}}{2} + E_{fr}\frac{W_{sr}}{2} + K'_{\varphi}\left[(1+E_{af}N_{\alpha f})\frac{C_{\gamma f}\Gamma_f}{C_{\alpha f}} + (1-E_{ar}N_{\alpha r})\frac{C_{\gamma r}(-\Gamma_r)}{C_{\alpha r}} + E_{sf} + E_{sr}\right]\right\}} \qquad (7)$$

The bicycle model assumes that the tires are operating in the linear range of performance where the rate of change of cornering stiffness with vertical load is relatively constant, therefore axle cornering stiffness is relatively constant

3/ Where are the lateral load transfer terms? (continued)

Assuming a vehicle with a static load of 4000N on both the left and right front tires, we can determine the **axle** cornering stiffness from tires for the bicycle model from the plot of tire cornering stiffness versus vertical load



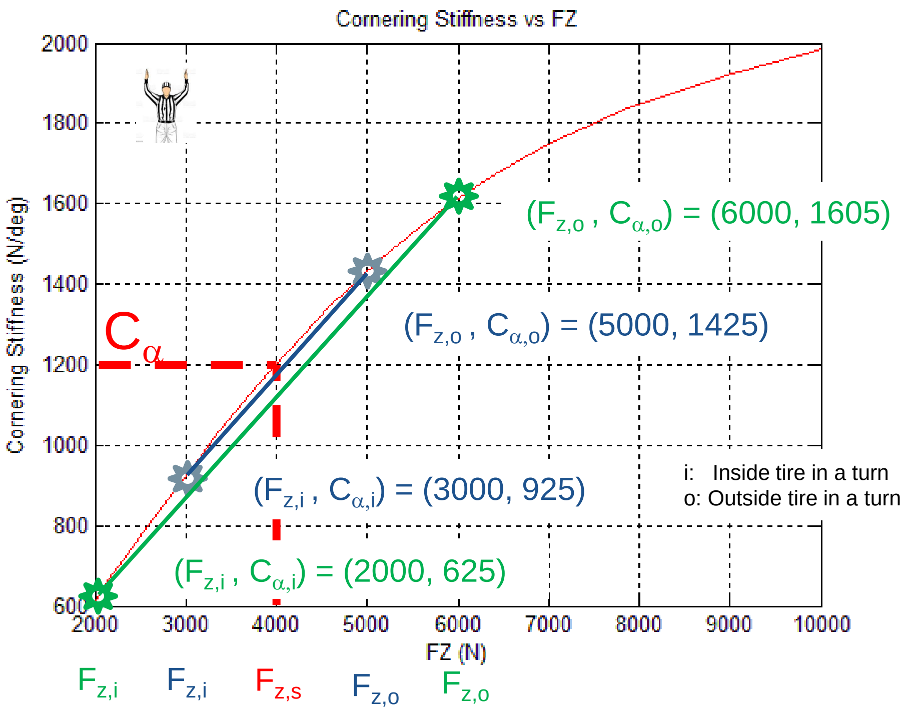
Front axle cornering stiffness from tires:

"Bicycle Model" – No lateral load transfer

$C_{\alpha,a} = 2^* C_\alpha = 2 * 1200 = 2400$ N/deg

3/ Where are the lateral load transfer terms? (continued)

Now, lets consider "linear range" operation, with relatively low lateral load transfer and "non-linear range" operation with relatively high lateral load transfer



Front axle cornering stiffness from tires:

"Bicycle Model" – No lateral load transfer

$C_{\alpha,a} = 2^* C_\alpha = 2 * 1200 = 2400$ N/deg

"Linear Range" – Low lateral load transfer

$C_{\alpha,a} = C_{\alpha,i} + C_{\alpha,o} = 1425 + 925 = 2350$ N/deg

$C_{\alpha,a} \sim C_{\alpha,a}$

"Non Linear Range" – High lateral load transfer

$C_{\alpha,a} = C_{\alpha,i} + C_{\alpha,o} = 1605 + 625 = 2230$ N/deg

$C_{\alpha,a} < C_{\alpha,a}$

Conclusion: The bicycle model assumption of no lateral load transfer is valid in the "linear range" of operation

Gillespie’s book, “Fundamentals of Vehicle Dynamics”, Chapter 6 gives a good presentation on the understeer budget equations, similar to Bundorf but with a couple of exceptions:

- Equation for the effect of aligning torque effect on the whole vehicle is ‘stand alone’ and a compact form of what Bundorf presented
- Adds an equation for contribution of upper steering system compliance (steering column, intermediate shaft, power steering torsion bar) to the lumped cornering compliance.
  - If you decide to include this in your understeer budget, be sure to use opposed lateral force and opposed aligning torque data rather than parallel. Otherwise, you will double count the effect of the upper steering compliance

Calculating the Understeer Budget

In 1972, Nedley and Wilson, also from General Motors, published their SAE paper “A New Laboratory Facility for Measuring Vehicle Parameters Affecting Understeer and Brake Steer”. In addition to describing GM’s new K&C test machine, this paper:

- Expanded on Bundorf’s equation to include camber compliance
- Wrote equations in generic terms so we can easily separate front and rear effects

We will use Bundorf’s equations (SAE paper 670078) and the camber compliance equation to calculate the understeer budget for a production EV sedan.

Calculating the Understeer Budget

<u>An important note about Understeer Budget sign convention:</u>

Bundorf, in 670078, uses a sign convention where an understeering component of the budget is termed positive, and an oversteering component is termed negative, regardless if it is front or rear axle.

In order to maintain continuity with the bicycle model sign convention (ISO), we will assign the following convention:

- Front axle: Understeer = Positive
- Rear axle: Oversteer = Positive

In this case, positive terms are generating sideslip angle in the same direction, regardless of their position on the vehicle (front or rear axle)

Calculating the Understeer Budget

Vehicle Parameters:

Parameter	Symbol	Units	Value
Front axle weight at 2-pass load	W_f	N	9161
Rear axle weight at 2-pass load	W_r	N	9615
Front axle unsprung weight at 2 pass load (1)	$W_{us,f}$	N	916
Rear axle unsprung weight at 2 pass load (1)	$W_{us,f}$	N	962
Wheelbase	L	m	2.876
Horizontal distance from Cg to front axle (calculated)	a	m	1.473
Horizontal distance from Cg to rear axle (calculated)	b	m	1.403
Vehicle roll gradient (2)	k'_φ	deg/g	2.71
Overall steering ratio	i	:1	11.7

(1) Using RoT that the axle unsprung weight is ~10% of the total axle weight for independent suspensions
(2) Steady state roll gradient from 75kph swept steer (constant speed understeer) test

Calculating the Understeer Budget

Tire Parameters (values for a single tire, at 2-pass corner weight):

Parameter	Symbol	Units	Value
Frt cornering stiffness	$C_{\alpha,f}$	N/deg	1437
Rr cornering stiffness	$C_{\alpha,r}$	N/deg	1507
Frt aligning torque stiffness	$N_{\alpha,f}$	Nm/deg	34
Rr aligning torque stiffness	$N_{\alpha,r}$	Nm/deg	38
Frt camber stiffness	$C_{\gamma,f}$	N/deg	114
Rr camber stiffness	$C_{\gamma,r}$	N/deg	122

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K&C Parameters

- Data used in this example courtesy of Morse Measurements
- Vehicle tested on their Anthony Best Dynamics K&C test machine
- Sign convention is ISO standard

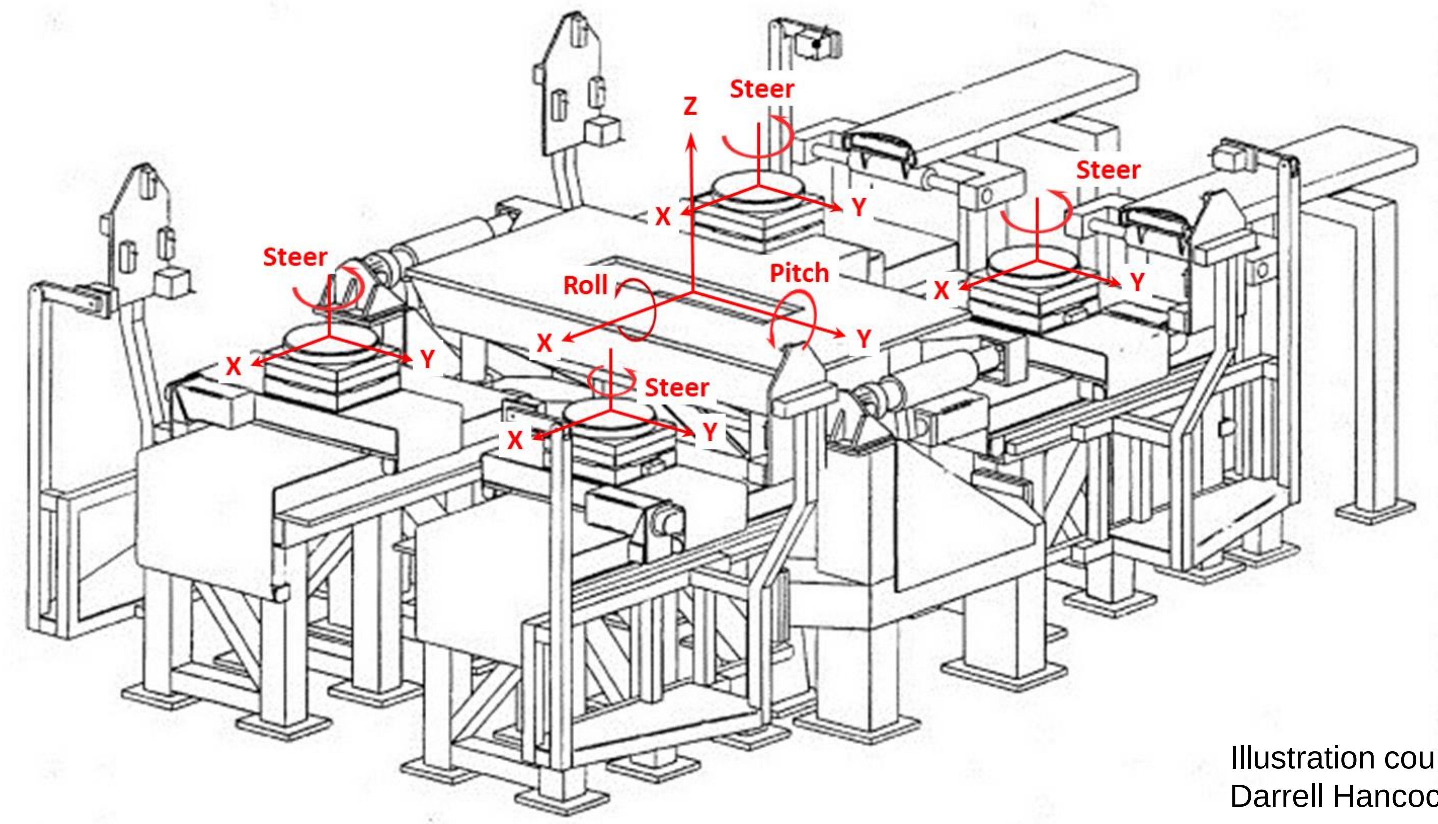

Illustration courtesy of Darrell Hancock

Calculating the Understeer Budget

K&C Parameters (average of left and right wheel properties):

Parameter	Symbol	Units	KC value	U/S or O/S effect?	KC value Understeer Budget sign convention
Frt lateral force compliance steer	$E_{f,f}$	deg/N	-5.80E-05	U/S	5.80E-05
Rr lateral force compliance steer	$E_{f,r}$	deg/N	1.04E-05	U/S	-1.04E-05
Frt lateral force compliance camber	$E_{g,f}$	deg/N	1.03E-04	U/S	1.03E-04
Rr lateral force compliance camber	$E_{g,r}$	deg/N	1.00E-04	O/S	1.00E-04
Frt aligning torque compliance steer	$E_{a,f}$	deg/Nm	2.35E-03	U/S	2.35E-03
Rr aligning torque compliance steer	$E_{a,r}$	deg/Nm	5.01E-04	O/S	5.01E-04

Remember our sign convention:

- Front axle: Understeer = Positive
- Rear axle: Oversteer = Positive

K&C Parameters (average of left and right wheel properties):

Parameter	Symbol	Units	KC value	U/S or O/S effect?	KC value Understeer Budget sign convention
Frt roll steer	$E_{s,f}$	deg/deg	-0.11	U/S	0.11
Rr roll steer	$E_{s,r}$	deg/deg	0.03	U/S	-0.03
Frt roll camber angle gain	Γ_f	deg/deg	-0.68		
Rr roll camber angle gain	Γ_r	deg/deg	-0.74		
Frt roll inclination angle gain (calc) (1)	Γ_f'	deg/deg	0.32	U/S	0.32
Rr roll inclination angle gain (calc) (1)	Γ_r'	deg/deg	0.26	O/S	0.26

Remember our sign convention:

- Front axle: Understeer = Positive
- Rear axle: Oversteer = Positive

(1) Roll inclination angle gain = 1+ roll camber gain

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Weight and Tire Effect [1]

$$D_{wt,f} = \left[\frac{W_f}{2 * C_{\alpha f}}\right] * \left[1 + \frac{N_{\alpha r}}{b * C_{\alpha r}}\right]$$

$$D_{wt,r} = \left[\frac{W_r}{2 * C_{\alpha r}}\right] * \left[1 - \frac{N_{\alpha f}}{a * C_{\alpha f}}\right]$$

W_f = 9161 N
W_r = 9615 N
a = 1.473 m
b = 1.403 m
$C_{\alpha,f}$ = 1437 N/deg
$C_{\alpha,r}$ = 1507 N/deg
$N_{\alpha,f}$ = 34 N-m/deg
$N_{\alpha,r}$ = 38 N-m/deg

Example vehicle:

$$D_{wt,f} = \left[\frac{9161}{2*1437}\right] * \left[1 + \frac{38}{1.403*1507}\right] = 3.24 \text{ deg/g} \Rightarrow \text{U/S}$$

$$D_{wt,r} = \left[\frac{9615}{2*1507}\right] * \left[1 - \frac{34}{1.473*1437}\right] = 3.14 \text{ deg/g} \Rightarrow \text{O/S}$$

(1) "W" does not include aero lift forces. If you have an aero sensitive car, might want to include them

Vehicle effect:

$$K_{wt} = D_{wt,f} - D_{wt,r} = 3.244 - 3.139 = 0.10 \text{ deg/g} \Rightarrow \text{U/S}$$

Lateral Force Compliance Steer Effect

$$D_{f,f} = \frac{W_{s,f}}{2} * E_{f,f}$$

$$D_{f,r} = \frac{W_{s,r}}{2} * E_{f,r}$$

$W_{s,f}$ = W_f - $W_{us,f}$ = 8243 N
$W_{s,r}$ = W_r - $W_{us,r}$ = 8654 N
$E_{f,f}$ = 5.80E-05 deg/N
$E_{f,r}$ = -1.04E-05 deg/N

Example vehicle:

$$D_{f,f} = 5.80e - 05 \ * \frac{8243}{2} = 0.24 \text{ deg/g => U/S}$$

$$D_{f,r} = -1.04e - 05 \ * \frac{8654}{2} = \text{- } 0.05 \text{ deg/g => U/S}$$

Vehicle Effect:

$$K_f = D_{f,f} - D_{f,r} = 0.24 - (\text{-}0.05) = 0.29 \text{ deg/g => U/S}$$

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Lateral Force Compliance Camber Effect

$$D_{g,f} = \frac{W_{s,f}}{2} * E_{g,f} * \frac{C_{\gamma,f}}{C_{\alpha,f}}$$

$$D_{g,r} = \frac{W_{s,r}}{2} * E_{g,r} * \frac{C_{\gamma,r}}{C_{\alpha,r}}$$

$W_{s,f} = W_f - W_{us,f}$ = 8243 N
$W_{s,r} = W_r - W_{us,r}$ = 8654 N
$E_{g,f}$ = 1.03E-04 deg/N
$E_{g,r}$ = 1.00E-04 deg/N
$C_{\alpha,f}$ = 1437 N/deg
$C_{\alpha,r}$ = 1507 N/deg
$C_{\gamma,f}$ = 114 N/deg
$C_{\gamma,r}$ = 122 N/deg

Example vehicle:

$$D_{g,f} = \frac{8243}{2} * 1.03e-04 * \frac{114}{1437} = 0.03 \text{ deg/g} => \text{U/S}$$

$$D_{g,r} = \frac{8654}{2} * 1.00e-04 * \frac{122}{1507} = 0.04 \text{ deg/g} => \text{O/S}$$

Vehicle Effect:

$$K_g = D_{g,f} - D_{g,r} = 0.03 - 0.04 = -0.01 \text{ deg/g} => \text{O/S}$$

Aligning Torque Compliance Steer Effect

$$D_{a,f} = \left(\frac{W_f}{2 * C_{\propto f}}\right) * (E_{a,f} * N_{\propto f}) * \left[1 + \frac{N_{\alpha r}}{b * C_{\alpha r}}\right]$$

$$D_{a,r} = \left(\frac{W_r}{2 * C_{\propto r}}\right) * (E_{a,r} * N_{\propto r}) * \left[1 - \frac{N_{\alpha f}}{a * C_{\alpha f}}\right]$$

W_f = 9161 N
W_r = 9615 N
a = 1.473 m
b = 1.403 m
$C_{\alpha,f}$ = 1437 N/deg
$C_{\alpha,r}$ = 1507 N/deg
$N_{\alpha,f}$ = 34 N-m/deg
$N_{\alpha,r}$ = 38 N-m/deg
$E_{a,f}$ = 0.0024 deg/N-m
$E_{a,r}$ = 0.0005 deg/N-m

Example vehicle:

$$D_{a,f} = \left(\frac{9161}{2*1437}\right) * (0.0024 * 34) * \left[1 + \frac{38}{1.403*1507}\right] = 0.26 \text{ deg/g} \Rightarrow \text{U/S}$$

$$D_{a,r} = \left(\frac{9615}{2*1507}\right) * (0.0005 * 38) * \left[1 - \frac{34}{1.473*1437}\right] = 0.05 \text{ deg/g} \Rightarrow \text{O/S}$$

Vehicle Effect:

$$K_{at} = D_{a,f} - D_{a,r} = 0.26 - 0.05 = 0.21 \text{ deg/g} \Rightarrow \text{U/S}$$

Kinematic Roll Steer Effect

k'_{φ} = 2.71 deg/g
$E_{s,f}$ = 0.11 deg/deg
$E_{s,r}$ = -0.03 deg/deg

$$D_{s,f} = k'_{\varphi} * E_{s,f}$$

$$D_{s,r} = k'_{\varphi} * E_{s,r}$$

Example vehicle:

$$D_{s,f} = 2.71 * 0.11 = 0.29 \text{ deg/g} \implies \text{U/S}$$

$$D_{s,r} = 2.71 * -0.03 = -0.07 \text{ deg/g} \implies \text{U/S}$$

Vehicle Effect:

$$K_s = D_{s,f} - D_{s,r} = 0.29 - (-0.07) = 0.36 \text{ deg/g} \implies \text{U/S}$$

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Kinematic Roll Camber Effect

$$D_{gp,f} = k'_{\varphi} * \left[\left(1 + E_{a,f} * N_{a,f}\right) * \frac{C_{\gamma,f} * \Gamma'_f}{C_{\alpha,f}}\right]$$

$$D_{gp,r} = k'_{\varphi} * \left[\left(1 - E_{a,r} * N_{a,r}\right) * \frac{C_{\gamma,r} * \Gamma'_r}{C_{\alpha,r}}\right]$$

k'_{φ} = 2.71 deg/g
$E_{a,f}$ = 0.0024 deg/N-m
$E_{a,r}$ = 0.0005 deg/N-m
Γ_f' = 0.32 deg/deg
Γ_r' = 0.26 deg/deg
$C_{\alpha,f}$ = 1437 N/deg
$C_{\alpha,r}$ = 1507 N/deg
$N_{\alpha,f}$ = 34 N-m/deg
$N_{\alpha,r}$ = 38 N-m/deg
$C_{\gamma,f}$ = 114 N/deg
$C_{\gamma,r}$ = 122 N/deg

Example vehicle:

$$D_{gp,f} = 2.71 * \left[(1 + (0.0024 * 34)) * \frac{114 * 0.32}{1437}\right] = 0.07 \text{ deg/g} \Rightarrow \text{U/S}$$

$$D_{gp,r} = 2.71 * \left[(1 - (0.0005 * 38)) * \frac{122 * 0.26}{1507}\right] = 0.06 \text{ deg/g} \Rightarrow \text{O/S}$$

Vehicle Effect:

$$K_{gp} = D_{gp,f} - D_{gp,r} = 0.07 - 0.06 = 0.01 \text{ deg/g} \Rightarrow \text{U/S}$$

Calculating the Understeer Budget

Component	Front +U/S	Rear +O/S	Net (F-R)	Units
Weight and Tire Effect	3.24	3.14	0.10	deg/g
Lateral Force Compliance Steer Effect	0.24	-0.05	0.29	deg/g
Lateral Force Compliance Camber Effect	0.03	0.04	-0.01	deg/g
Aligning Torque Compliance Steer Effect	0.26	0.05	0.21	deg/g
Kinematic Roll Steer Effect	0.29	-0.07	0.36	deg/g
Kinematic Roll Camber Effect	0.07	0.06	0.01	deg/g
Cornering Compliance	4.13	3.17		deg/g
Understeer Gradient			0.96	deg/g

Outline

We can plug the front and rear cornering compliances into the bicycle model transfer functions to get an estimate of steady state and transient planar handling performance

$$\frac{r}{\delta_{sw}} = \frac{1}{i} * \frac{\left[\frac{D_r V_v^2}{57.3g}\right]s+1}{\left[\frac{\frac{(\frac{k_{z,v}^2}{ab})V_v^2 D_f D_r}{(57.3g)^2}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s^2 + \left[\frac{\frac{V_v(D_f-D_r)}{57.3g}+\frac{V_v(\frac{k_{z,v}^2}{ab}-1)(aD_f+bD_r)}{57.3g(a+b)}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s+1} * \left(\frac{r}{\delta_{sw}}\right)_{ss}$$

$$\left(\frac{r}{\delta_{sw}}\right)_{ss} = \frac{1}{i} * \frac{\frac{V_v}{a+b}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}$$

Steady State Yaw Gain

$$\frac{\beta_{vm}}{\delta_{sw}} = \frac{1}{i} * \frac{\frac{\frac{D_r V_v}{57.3g}(1+\frac{2k_{z,v}^2}{a(a+b)}}{1-\frac{2D_r V_v^2}{57.3g(a+b)}}}{\left[\frac{\frac{(\frac{k_{z,v}^2}{ab})V_v^2 D_f D_r}{(57.3g)^2}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s^2 + \left[\frac{\frac{u(D_f-D_r)}{57.3g}+\frac{V_v(\frac{k_{z,v}^2}{ab}-1)(aD_f+bD_r)}{57.3g(a+b)}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s+1} * \left(\frac{\beta_{vm}}{\delta_{sw}}\right)_{ss}$$

$$\left(\frac{\beta_{vm}}{\delta_{sw}}\right)_{ss} = \frac{1}{i} * \frac{\frac{1}{2}-\frac{D_r V_v^2}{57.3g(a+b)}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}$$

Steady State Sideslip Angle Gain

$$\frac{a_{ym}}{\delta_{sw}} = \frac{1}{i} * \frac{\left[\frac{D_r}{57.3g}\left(\frac{k_{z,v}^2}{a}+\frac{a+b}{2}\right)\right]s^2+\left[\frac{a+b}{2V_v}\right]s+1}{\left[\frac{\frac{(\frac{k_{z,v}^2}{ab})V_v^2 D_f D_r}{(57.3g)^2}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s^2 + \left[\frac{\frac{u(D_f-D_r)}{57.3g}+\frac{V_v(\frac{k_{z,v}^2}{ab}-1)(aD_f+bD_r)}{57.3g(a+b)}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s+1} * \left(\frac{a_{ym}}{\delta_{sw}}\right)_{ss}$$

$$\left(\frac{a_{ym}}{\delta_{sw}}\right)_{ss} = \frac{1}{i} * \frac{\frac{V_v^2}{a+b}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}$$

Steady State Lat Acc Gain

Yaw rate / Steering wheel angle steady state gain

V_f = 20.83m/s (75kph)
a = 1.473 m
b = 1.403 m
$D_{,f}$ = 4.13 deg/g
$D_{,r}$ = 3.17 deg/g
i = 11.7 (:1)
g = 9.81 m/s^2 (/g)

$$\left(\frac{r}{\delta_{sw}}\right)_{ss} = \frac{1}{i} * \frac{\frac{V_v}{a+b}}{1+\frac{V_v^2\left(D_f - D_r\right)}{57.3g(a+b)}} = \frac{1}{11.7} * \frac{\frac{20.83}{1.473+1.403}}{1+\frac{(20.83)^2 * (4.13-3.17)}{57.3 * 9.81 * (1.473+1.403)}}$$

$$\left(\frac{\boldsymbol{r}}{\boldsymbol{\delta_{sw}}}\right)_{\boldsymbol{ss}} = 0.49\, rad/s/rad = \mathbf{0.49}\, \boldsymbol{deg/s/deg}$$

Sideslip angle / Steering wheel angle steady state gain

$$\left(\frac{\beta_{vm}}{\delta_{sw}}\right)_{ss} = \frac{1}{i} * \frac{\frac{1}{2} - \frac{D_r V_v^2}{57.3g(a+b)}}{1+\frac{V_v^2\left(D_f - D_r\right)}{57.3g(a+b)}} = \frac{1}{11.7} * \frac{\frac{1}{2} - \frac{3.17 * (20.83)^2}{57.3 * 9.81 * (1.473+1.403)}}{1+\frac{(20.83)^2 * (4.13-3.17)}{57.3 * 9.81 * (1.473+1.403)}}$$

$$\left(\frac{\boldsymbol{\beta_{vm}}}{\boldsymbol{\delta_{sw}}}\right)_{\boldsymbol{ss}} = -0.02\frac{rad}{rad} = \mathbf{-0.02}\, \boldsymbol{deg/deg}$$

Lateral acceleration / Steering wheel angle steady state gain

$$\left(\frac{a_{ym}}{\delta_{sw}}\right)_{ss} = \frac{1}{i} * \frac{\frac{V_v^2}{a+b}}{1+\frac{V_v^2\left(D_f - D_r\right)}{57.3g(a+b)}} = \frac{1}{11.7} * \frac{\frac{(20.83)^2}{(1.473+1.403)}}{1+\frac{(20.83)^2 * (4.13-3.17)}{57.3 * 9.81 * (1.473+1.403)}}$$

$$\left(\frac{\boldsymbol{a_{ym}}}{\boldsymbol{\delta_{sw}}}\right)_{\boldsymbol{ss}} = 10.2\ m/s^2/rad = \mathbf{0.018}\, \boldsymbol{g/deg}$$

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9. **Correlation**
10. Summary

Correlation

Q: How well does is correlate to real world?”

A: Below is a comparison of predicted to physical test

Test	Metric	Units	Sim	Test	%Diff
Swept Steer @ 75kph	Frt. Cornering Compliance	deg/g	4.13	4.41	-7.0
Swept Steer @ 75kph	Rr. Cornering Compliance	deg/g	3.17	2.91	7.6
Swept Steer @ 75kph	Understeer Gradient	deg/g	0.96	1.50	-35.3
Step Steer @ 75kph	Steady State Yaw Gain	(deg/s)/deg	0.49	0.42	16.7
Step Steer @ 75kph	Steady State Sideslip Gain	deg/deg	-0.02	-0.01	50.0
Step Steer @ 75kph	Steady State Lat. Acc. Gain	g/deg	0.018	0.017	5.9

“Out of the box”, correlation doesn’t look too bad. But why isn’t it exact?

Steady state gains are off because the cornering compliances (and understeer gradient) are off.

Metric	Sim	Test	%Diff
D_f	4.13	4.41	-7.0
D_r	3.17	2.91	7.6
$D_f - D_r$	0.96	1.50	-35.3

$$\frac{r}{\delta_{sw}}=\frac{1}{i}*\frac{\left[\frac{D_r V_v^2}{57.3g}\right]s+1}{\left[\frac{\frac{\left(\frac{k_{z,v}^2}{ab}\right)V_v^2 D_f D_r}{(57.3g)^2}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s^2+\left[\frac{\frac{V_v(D_f-D_r)}{57.3g}+\frac{V_v\left(\frac{k_{z,v}^2}{ab}-1\right)(aD_f+bD_r)}{57.3g(a+b)}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s+1}*\left(\frac{r}{\delta_{sw}}\right)_{ss}$$

$$\left(\frac{r}{\delta_{sw}}\right)_{ss}=\frac{1}{i}*\frac{\frac{V_v}{a+b}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}$$

Steady State Yaw Gain

$$\frac{\beta_{vm}}{\delta_{sw}}=\frac{1}{i}*\frac{\frac{\frac{D_r V_v}{57.3g}\left(1+\frac{2k_{z,v}^2}{a(a+b)}\right)}{1-\frac{2D_r V_v^2}{57.3g(a+b)}}}{\left[\frac{\frac{\left(\frac{k_{z,v}^2}{ab}\right)V_v^2 D_f D_r}{(57.3g)^2}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s^2+\left[\frac{\frac{u(D_f-D_r)}{57.3g}+\frac{V_v\left(\frac{k_{z,v}^2}{ab}-1\right)(aD_f+bD_r)}{57.3g(a+b)}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s+1}*\left(\frac{\beta_{vm}}{\delta_{sw}}\right)_{ss}$$

$$\left(\frac{\beta_{vm}}{\delta_{sw}}\right)_{ss}=\frac{1}{i}*\frac{\frac{1}{2}-\frac{D_r V_v^2}{57.3g(a+b)}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}$$

Steady State Sideslip Angle Gain

$$\frac{a_{ym}}{\delta_{sw}}=\frac{1}{i}*\frac{\left[\frac{D_r}{57.3g}\left(\frac{k_{z,v}^2}{a}+\frac{a+b}{2}\right)\right]s^2+\left[\frac{a+b}{2V_v}\right]s+1}{\left[\frac{\frac{\left(\frac{k_{z,v}^2}{ab}\right)V_v^2 D_f D_r}{(57.3g)^2}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s^2+\left[\frac{\frac{u(D_f-D_r)}{57.3g}+\frac{V_v\left(\frac{k_{z,v}^2}{ab}-1\right)(aD_f+bD_r)}{57.3g(a+b)}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}\right]s+1}*\left(\frac{a_{ym}}{\delta_{sw}}\right)_{ss}$$

$$\left(\frac{a_{ym}}{\delta_{sw}}\right)_{ss}=\frac{1}{i}*\frac{\frac{V_v^2}{a+b}}{1+\frac{V_v^2(D_f-D_r)}{57.3g(a+b)}}$$

Steady State Lat Acc Gain

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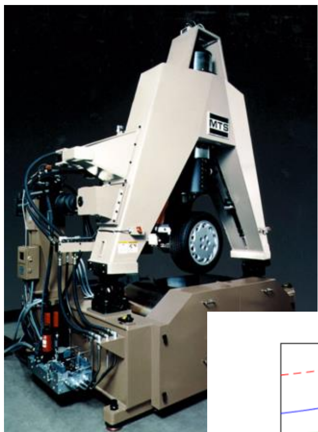
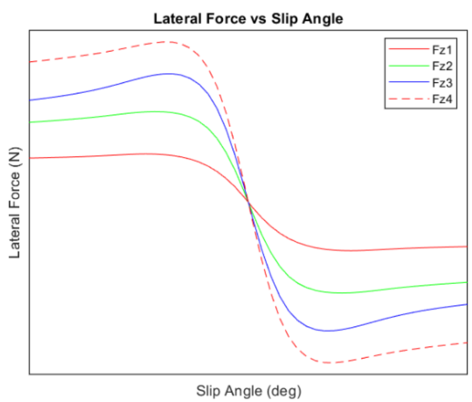
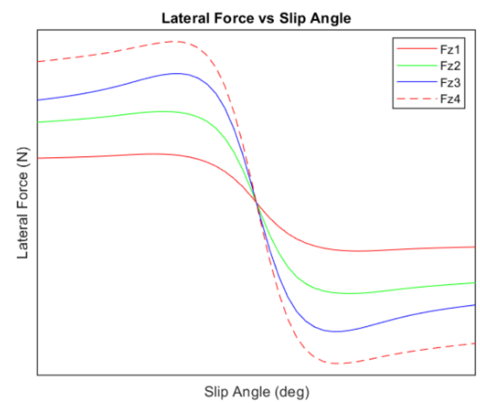
Correlation

Q: Why are the cornering compliances not perfectly correlated?
A:

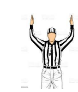
Metric	Sim	Test	%Diff
D_f	4.10	4.41	-7.0
D_r	3.13	2.91	7.6
$D_f - D_r$	0.97	1.50	-35.3







Tires are tested on 80 grit sandpaper but driven on asphalt, concrete, gravel, etc…



Correlation

The famous statistician, George Box, said:

“Essentially, all models are wrong, but some are useful.”

“… The practical question is: how wrong do they have to be to not be useful?”

The bicycle model and understeer budget are useful when used in the “Systems Engineering V” to:

- Cascade targets from the vehicle level to the chassis system level
- Assemble the chassis systems and tire performance status to verify vehicle targets will be met

With repeated correlation exercises, the user will determine appropriate tire surface correction factors to improve first time through accuracy

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Summary

- The Systems Engineering V and first principals are a useful combination to cascade vehicle handling response to K&C and tire parameters
- According to the bicycle model, vehicle planar handling response is strongly influenced by the lumped cornering compliances
- The lumped cornering compliance is comprised of individual compliances due to
  - Weight and Tires
  - Suspension Kinematics
  - Suspension Compliances
- The understeer budget concept can be used to determine the relative contribution of tires, kinematics and compliances on vehicle understeer gradient

Summary

- The Systems Engineering V and first principals are a useful combination to predict vehicle handling response from K&C and tire parameters
- The correlation between first principals (simulation) and test can be improved by applying surface correction factors to the tire flat track force and moment data.

Backup

Bicycle Model Variables and Units

Variable	**Units**	**Definition**
β_{vm}	rad	Vehicle sideslip angle at vehicle midpoint
V_v	m/s	Vehicle Velocity
δ_{sw}	rad	Steering wheel angle
a_{ym}	m/s^2	Lateral acceleration at vehicle midpoint
i	:1	Overall steering ratio
r	rad/s	Yaw velocity (aka yaw rate)
a, b	m	Distance from vehicle center of gravity to front, rear axle
$k_{z,v}^2$	m^2	Yaw radius of gyration
$D_{f,}$ D_r	deg/g	Front rear cornering compliance
g	9.806 m/s^2	Acceleration due to gravity

Understeer Budget Variables and Units

Variable	Units	Definition
W_f, W_r	N	Front, rear vehicle weight
W_{sf}, W_{sr}	N	Front, rear sprung weight
$C_{\alpha f}$, $C_{\alpha r}$	N/deg	Front, rear tire cornering stiffness
$N_{\alpha f}$, $N_{\alpha r}$	Nm/deg	Front, rear tire aligning torque stiffness
$C_{\gamma f}$, $C_{\gamma r}$	N/deg	Front, rear tire camber stiffness
K'_{φ}	deg/g	Vehicle roll gradient
E_{sf}, E_{sr}	deg/deg	Front, rear roll steer coef.
Γ_f, Γ_r	deg/deg	Front, rear roll camber coef.
Γ'_f, Γ'_r	deg/deg	Front, rear roll inclination coef. ($\Gamma' = 1 + \Gamma$)
$E_{\delta af}$, $E_{\delta ar}$	deg/Nm	Front, rear aligning torque compliance steer
$E_{\delta ff}$, $E_{\delta fr}$	deg/N	Front, rear lateral force compliance steer
$E_{\gamma ff}$, $E_{\gamma fr}$	deg/N	Front, rear lateral force compliance camber

Additional References

“**Directional Control Dynamics of Automobile-Travel Trailer Combinations”**, R.T. Bundorf; 670099, SAE International; 1967

“**Vehicle Dynamics**”; J. R. Ellis, London Business Books, 1969

“**The Handling Properties of Light Trucks**”, D. J. Bickerstaff; 760710, SAE International; 1976

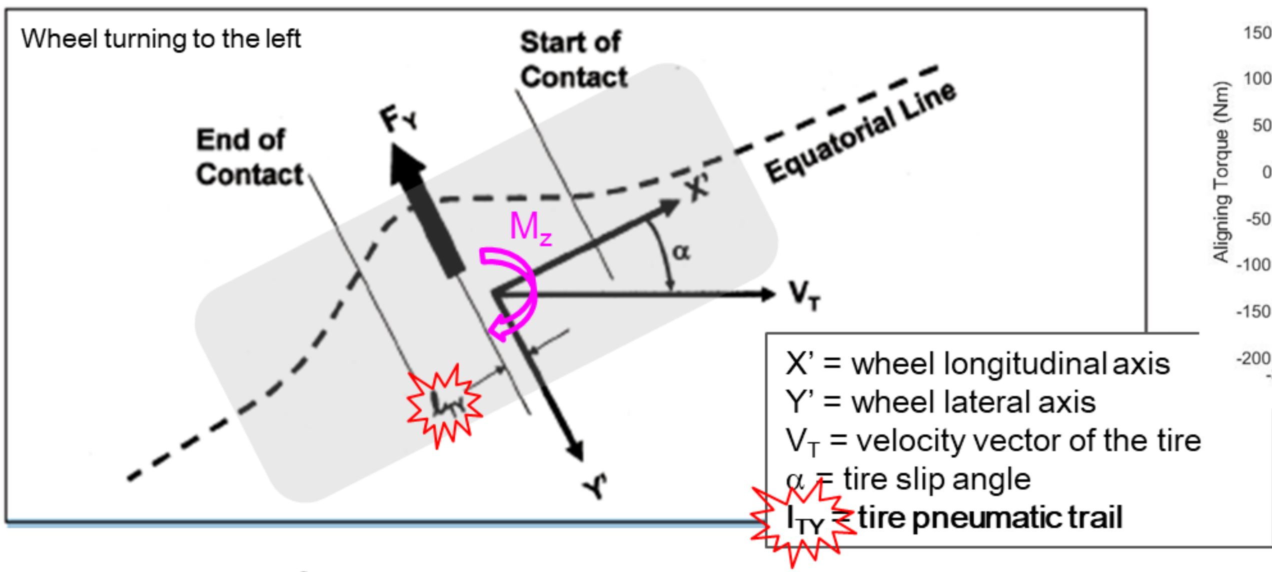
“**Fundamentals of Vehicle Dynamics, Revised Edition**”; T. D. Gillespie; R-506, SAE International; 2021

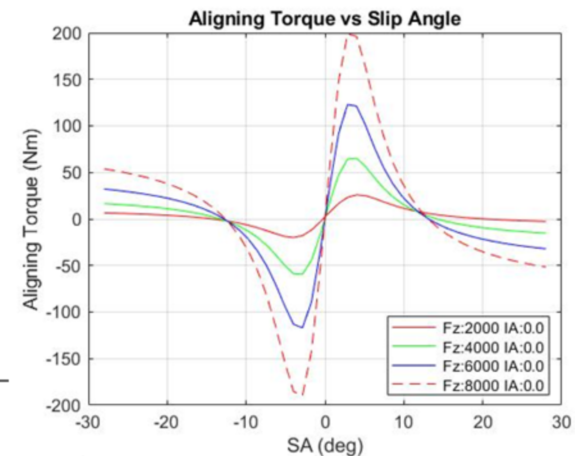
“**The Complex Cornering Compliance Theory and its Application to Vehicle Dynamics Characteristics**”; K. Tsuji and N. Totoki; 2002-01-1218, SAE International; 2002

Pneumatic trail

- The lateral force distribution in the tire contact patch is non-uniform
- The center of lateral force, for a free rolling tire (no longitudinal slip) is aft of the geometric center of the tire a distance called the '**pneumatic trail**'

Figure 8.3: String tire at a slip angle in contact with the road

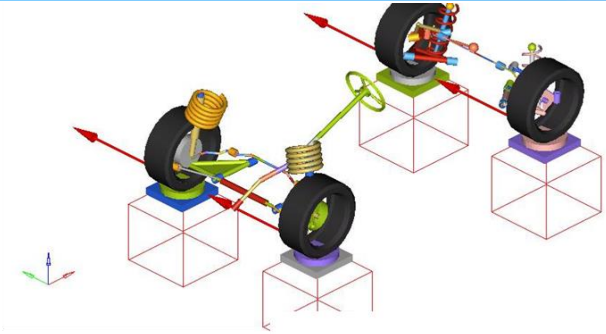




Exemplar data from a tire tested on Flat-Trac® machine (surface = 3Mite80 Clearback)

- The lateral force times the pneumatic trail creates an aligning torque, or more precisely, the tire **self-aligning torque**, M_z
  - This is one of the components in the vehicle system that causes a buildup in steering wheel torque when beginning a turn, and the return of the vehicle to straight ahead at the end of a turn

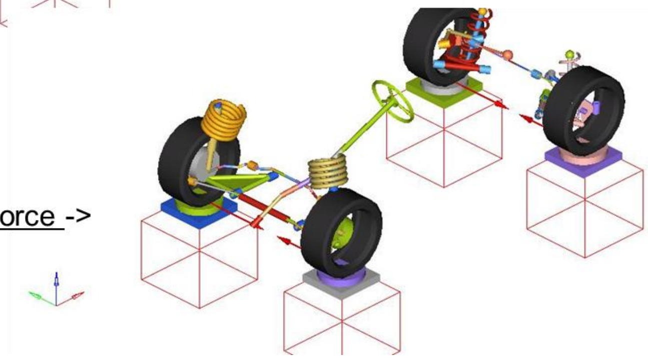
Parallel vs Opposed K&C Compliance Tests

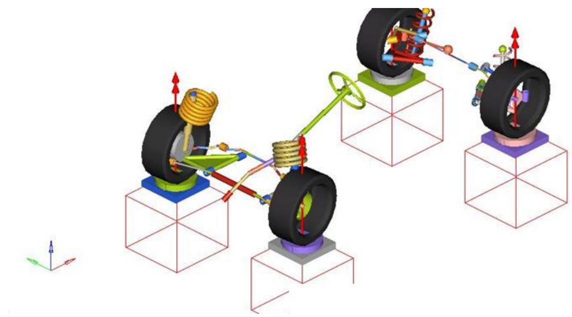


<- Parallel Lateral Force

Parallel (aka "aiding"):
Left and right applied forces or moments equal in magnitude and direction

Opposed Lateral Force ->

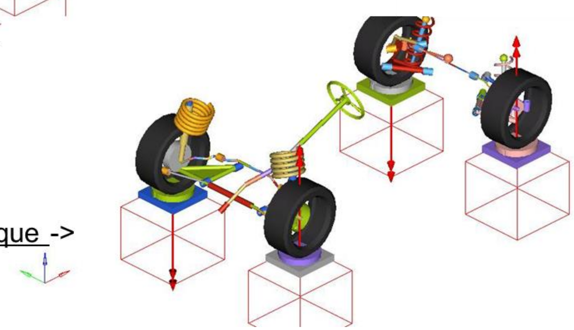




<- Parallel Aligning Torque

Opposed:
Left and right applied forces or moments equal in magnitude but opposite in direction

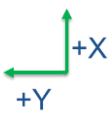
Opposed Aligning Torque ->



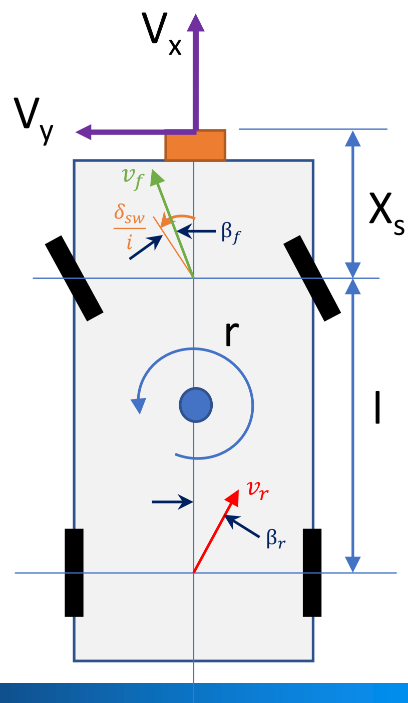
Axle Side Slip Angle Calculation

Case where the lateral velocity sensor is mounted to the front bumper at the centerline of vehicle

- $\beta_f = \tan^{-1}\left(\frac{V_y - X_s * r}{V_x}\right) - \frac{\delta_{sw}}{i}$
- $\beta_r = \tan^{-1}\left(\frac{V_y - (X_s + l) * r}{V_x}\right)$



ISO or SAE “Z-up” sign convention



β_f = *Front axle sideslip angle (rad)*

β_r = *Rear axle sideslip angle (rad)*

V_x = *Forward (longitudinal) speed* $\left(\frac{m}{s}\right)$

V_y = *Lateral velocity* $\left(\frac{m}{s}\right)$

X_S = *Longitudinal distance between the front axle and the lateral speed sensor (m; +sensor forward of front axle)*

l = *wheelbase (m)*

r = *yaw rate* $\left(\frac{rad}{s}\right)$

δ_{sw} = *Steering wheel angle (rad)*

i = *Overall steering ratio (:1)*

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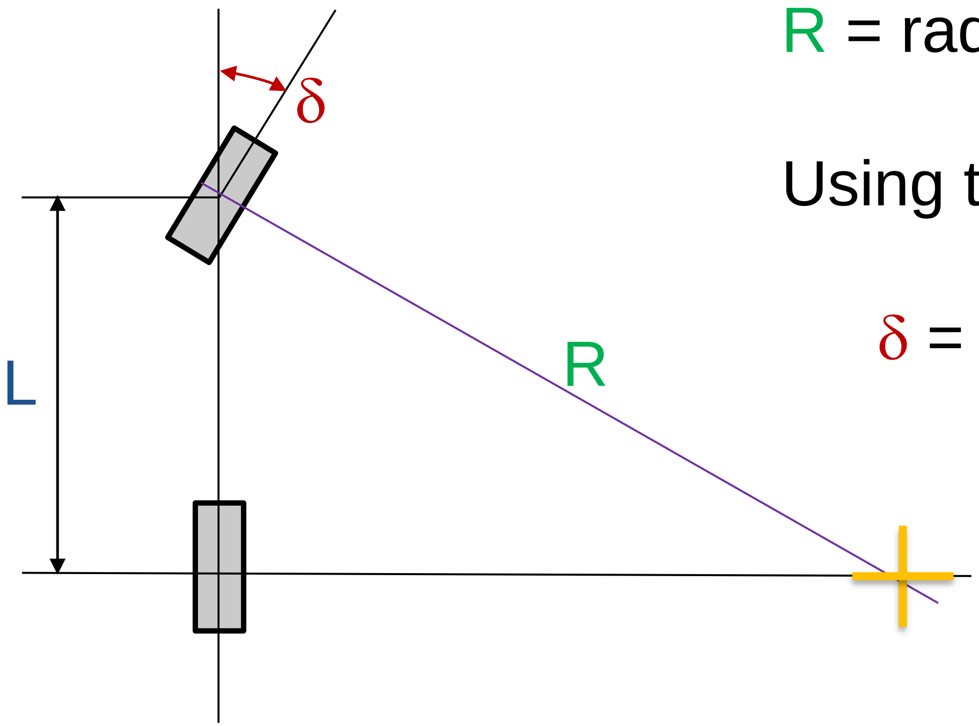
Kinematic Bicycle Model

L = wheelbase, m
δ = average front steer angle, radians
R = radius of turn, m



Using the property of similar triangles,

$$\delta = L / R \text{ (rad)}$$

Q: When do you transition from kinematic to dynamic model?
A: When the tires start to generate noticeable sideslip and the vehicle starts to build up lateral acceleration