

Introduction to Tire Forces and Moments

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About the author



Tim Drotar is currently a senior staff engineer in vehicle dynamics at Tesla. Prior to joining Tesla, he spent 30 years at Ford Motor Company where he specialized in chassis systems and vehicle dynamics for passenger cars and light trucks. Tim is a member of SAE, SCCA and The Tire Society. He holds a B.S. in Mechanical Engineering from Lawrence Technological University and a M.S. in Mechanical Engineering from the University of Michigan-Dearborn.

Tim also teaches the following classes for SAE:

- Advanced Vehicle Dynamics for Passenger Cars and Light Trucks
 - <https://www.sae.org/learn/content/c0415/>
- Fundamentals of Steering Systems
 - <https://www.sae.org/learn/content/c0716/>

Introduction

In this presentation, the basic lateral and longitudinal force behavior of tires will be discussed. We will study common parameters used to describe tire behavior as it relates to vehicle dynamics performance.

A good reference on tires is “The Pneumatic Tire”, available as an eBook that can be downloaded at no charge from the NHTSA website

https://one.nhtsa.gov/staticfiles/safercar/pdf/PneumaticTire_HS-810-561.pdf

Introduction

- Pneumatic tire concept was patented in England by R. Thomson in 1846
- It is a durable and inexpensive device with complicated composite construction of rubber, metal and synthetic components. Specialized adhesives bond multiple components together
- “ ... the ***most*** complicated and useful devices man makes. Without tires we'd all be travelling in street cars or on railroads, and none of us could live more than walking distance from a rail line...”

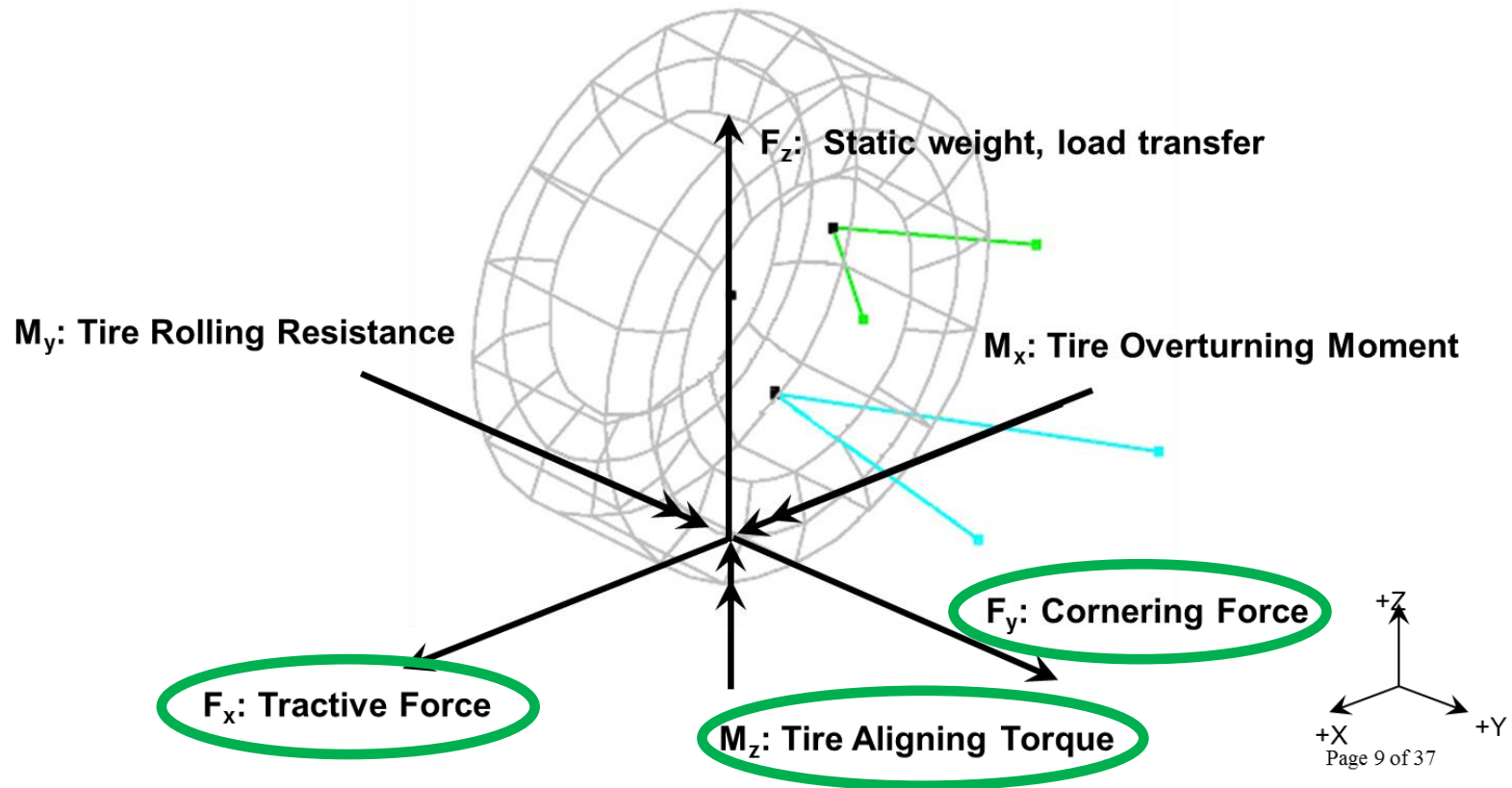
Introduction

- All forces (other than aero forces) that affect the motion of the vehicle come through the tires
- Tires have the biggest effect on determining vehicle's ride, steering and handling characteristics
- Tires have a number of conflicting requirements and affect many attributes
 - Steering, Handling, Braking/Acceleration
 - Ride
 - NVH
 - Durability
 - Wear
 - Fuel Economy
 - Package/Styling

In the following discussion we will focus on tire properties that affect steering/handling and braking

Introduction

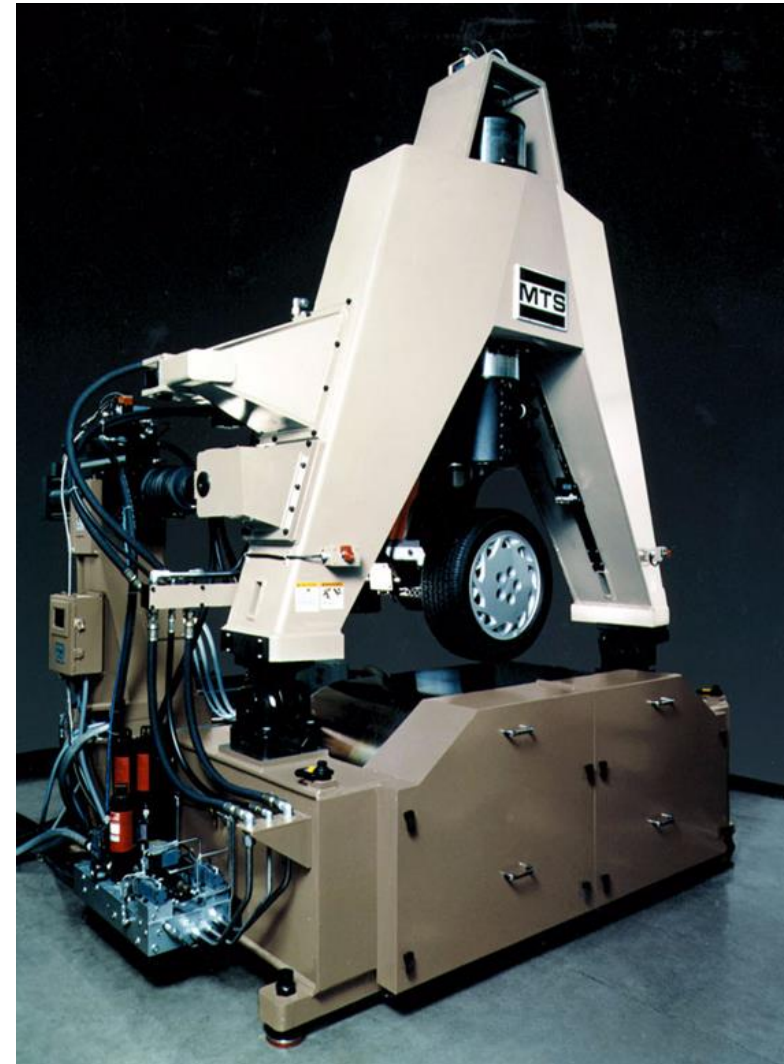
Driving down the road, tire forces and moments in all directions act on the vehicle simultaneously



This presentation will focus on cornering force, tractive force and aligning torque

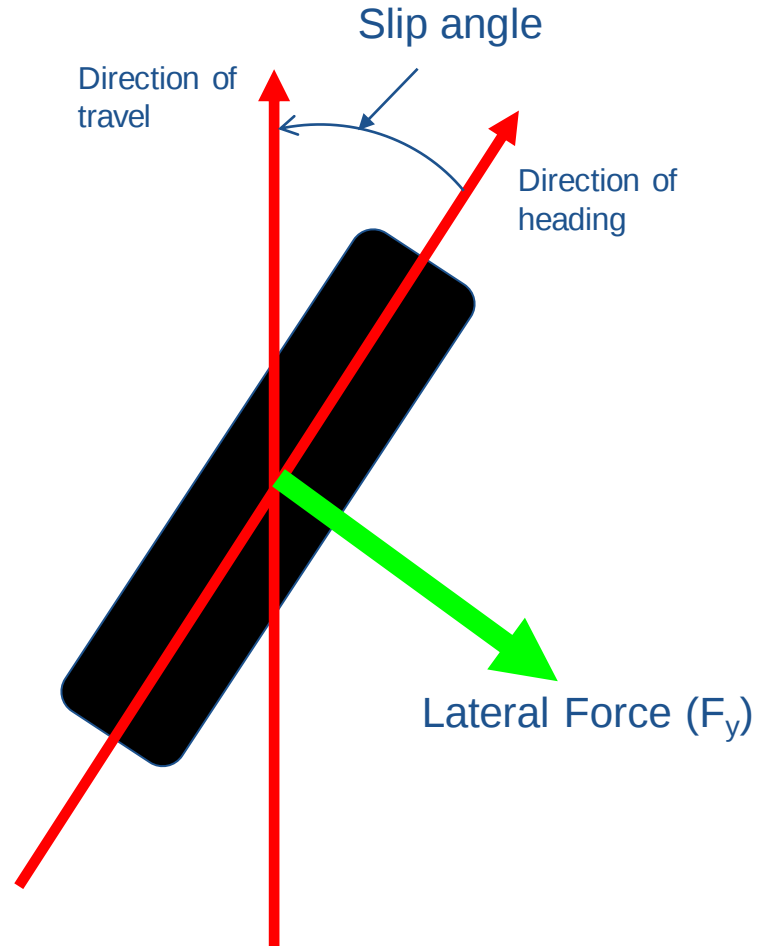
How Tire Forces and Moments are Measured

- Tires are commonly tested in a lab on a force and moment test rig.
- A machine commonly used by OEM's and tire manufacturers is the Flat Trac test machine from MTS
- The Flat Trac machine is like a large belt sander
- The belt is moving at a constant speed, the tire is placed in contact with the moving surface at a given vertical load and inclination angle. Load cells at the wheel center measure and the 3 forces and 3 moments being generated when
 - A steer angle is applied to the wheel -> Lateral Sweep Test
 - An acceleration or braking torque is applied to the wheel -> Longitudinal Slip Test

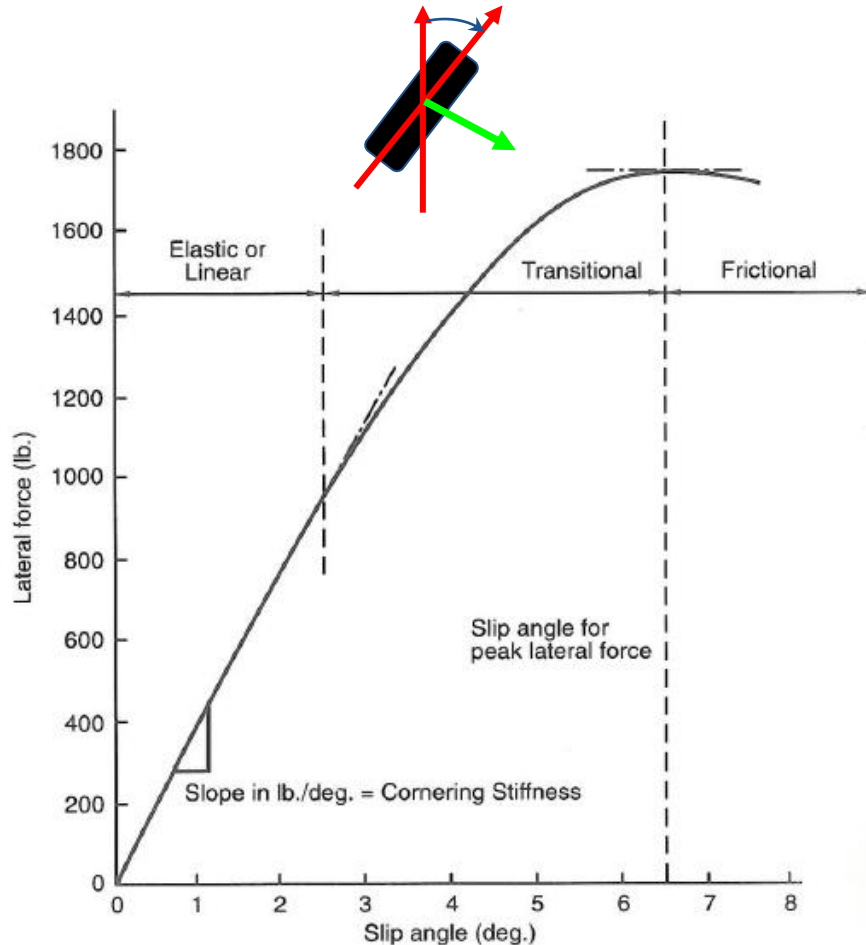


Tire slip angle

“the difference between where the tire is going and where it is headed”



Lateral Force



from Race Car Vehicle Dynamics, Milliken/Milliken

Linear region

- Tire behaves like a linear spring
- **Cornering stiffness** defined in this area (i.e. slope at 0 slip angle)

Transitional region

- Tire is moving from linear spring behavior to frictional behavior
- Cornering stiffness decreasing with more slip angle

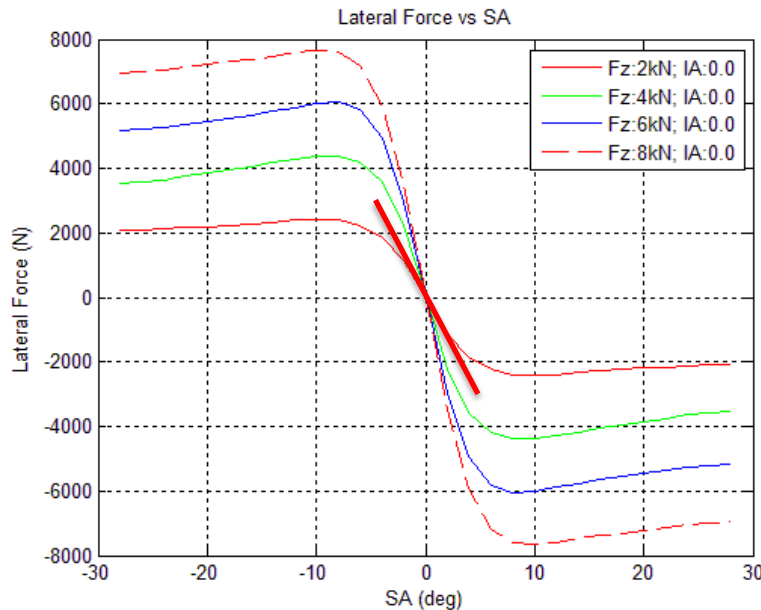
Frictional region

- Tire behaves more like rubber sliding on pavement
- Peak/saturated lateral force occur in this area ("**grip**")

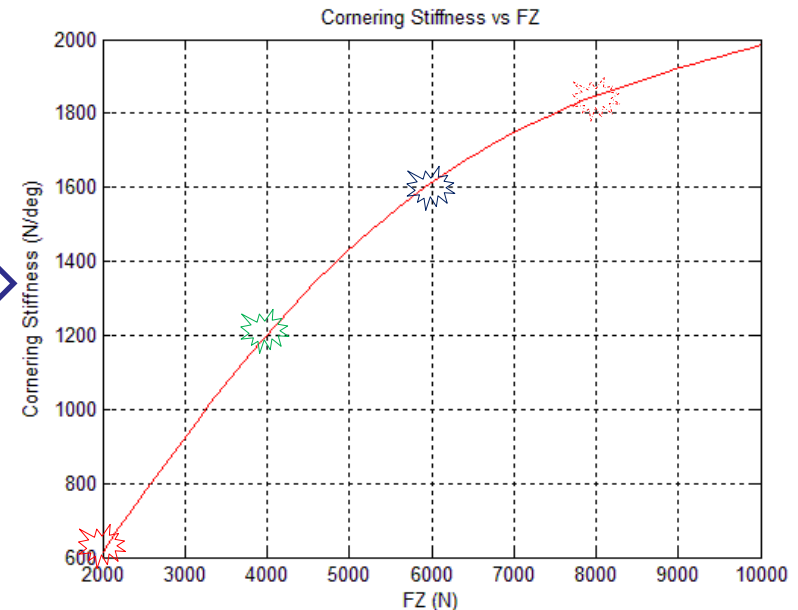
Lateral Force

Cornering Stiffness (Slope of F_y /Slip Angle curve at 0deg)

Exemplar data from a tire tested on Flat-Trac[®] machine (surface = 3Mite80 Clearback)



$$\left. \frac{dF_y}{dSA} \right|_{SA=0}$$

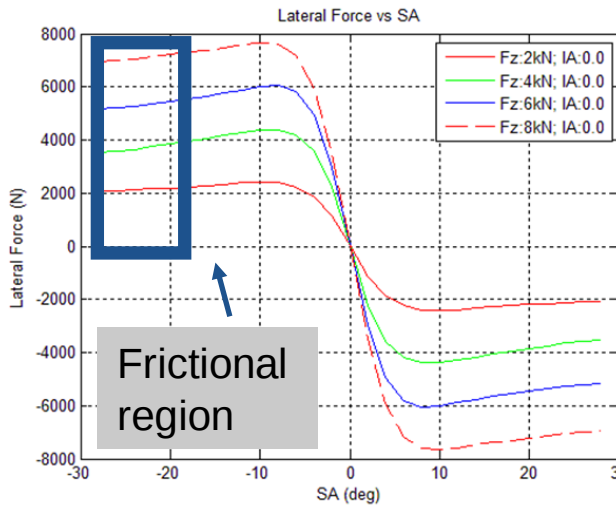
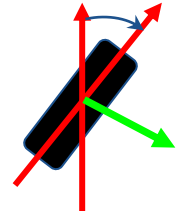


Cornering stiffness:

- Characterizes how effective ("stiff") the tire is in generating lateral force
- is a key tire parameter that affects vehicle response (A_y , yaw) to a steering input
- Changes with vertical load and inclination angle
- is affected by compound, tread block design, tire size, tire temperature, etc.

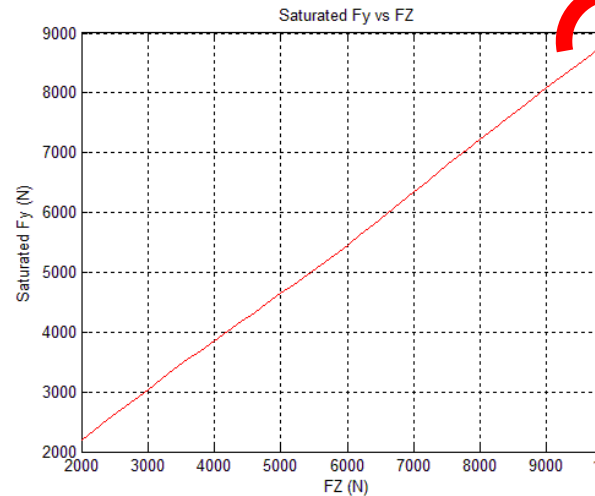
Lateral Force

Saturated Lateral Force (aka 'grip force')



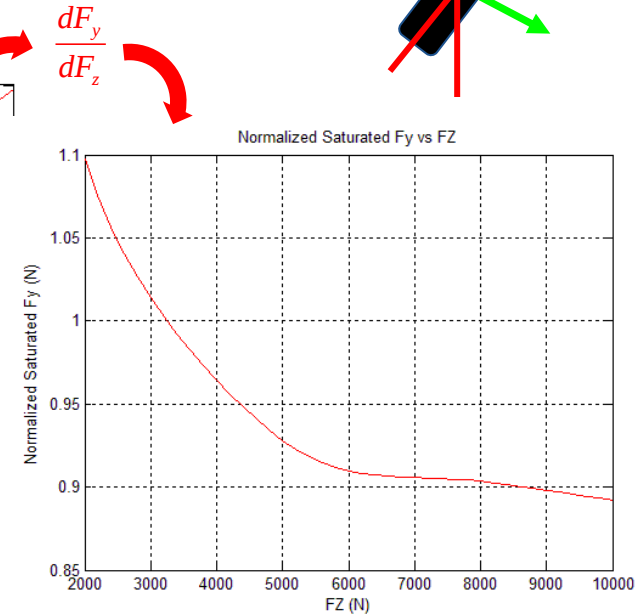
Saturated lateral force

- Tire rubber sliding on pavement
- "saturated" = more slip angle doesn't result in more Fy



Variation with Fz

- Saturated Fy increases with increasing Fz



Normalized Sat. Fy – "mu"

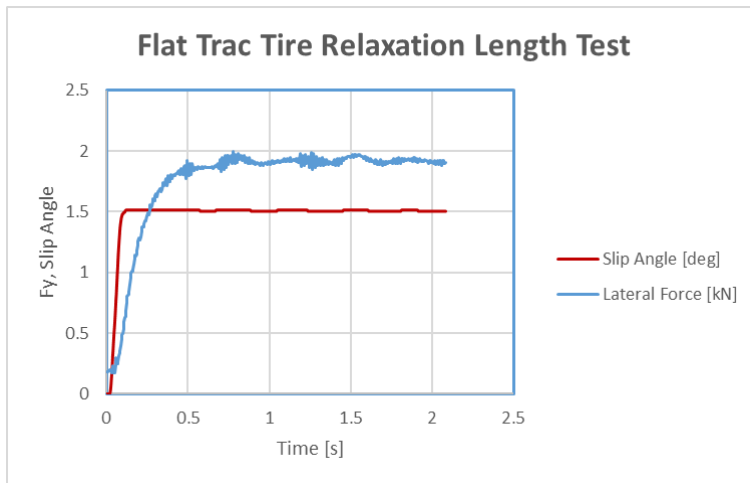
- Mu decreases with increasing Fz

- Measure of how "sticky" a tire is at high slip angles
- Usually expressed as a dimensionless value - coeff. of friction (mu)
- Key tire parameter that affects max. Ay, limit handling balance, etc.
- Affected by compound, tire size, tire temperature, etc.

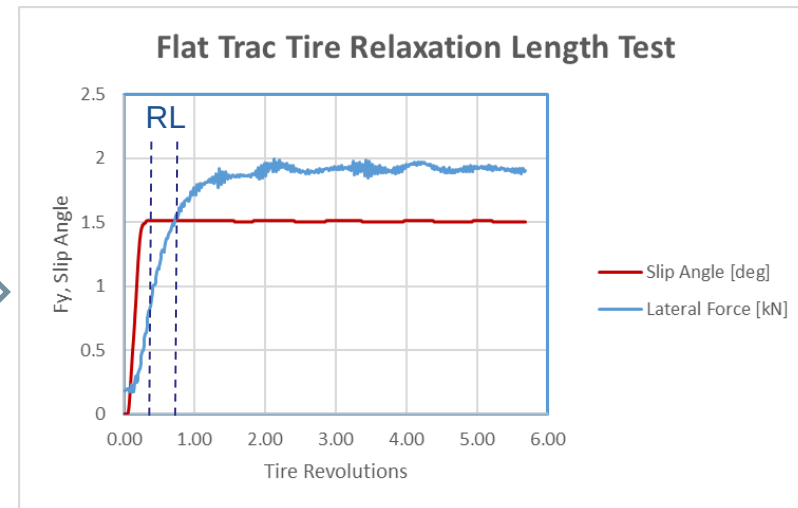
Lateral Force

Lateral Force Relaxation Length

- Distance travelled by a tire before it reaches a steady state lateral force after a step steer input is applied to the road wheel



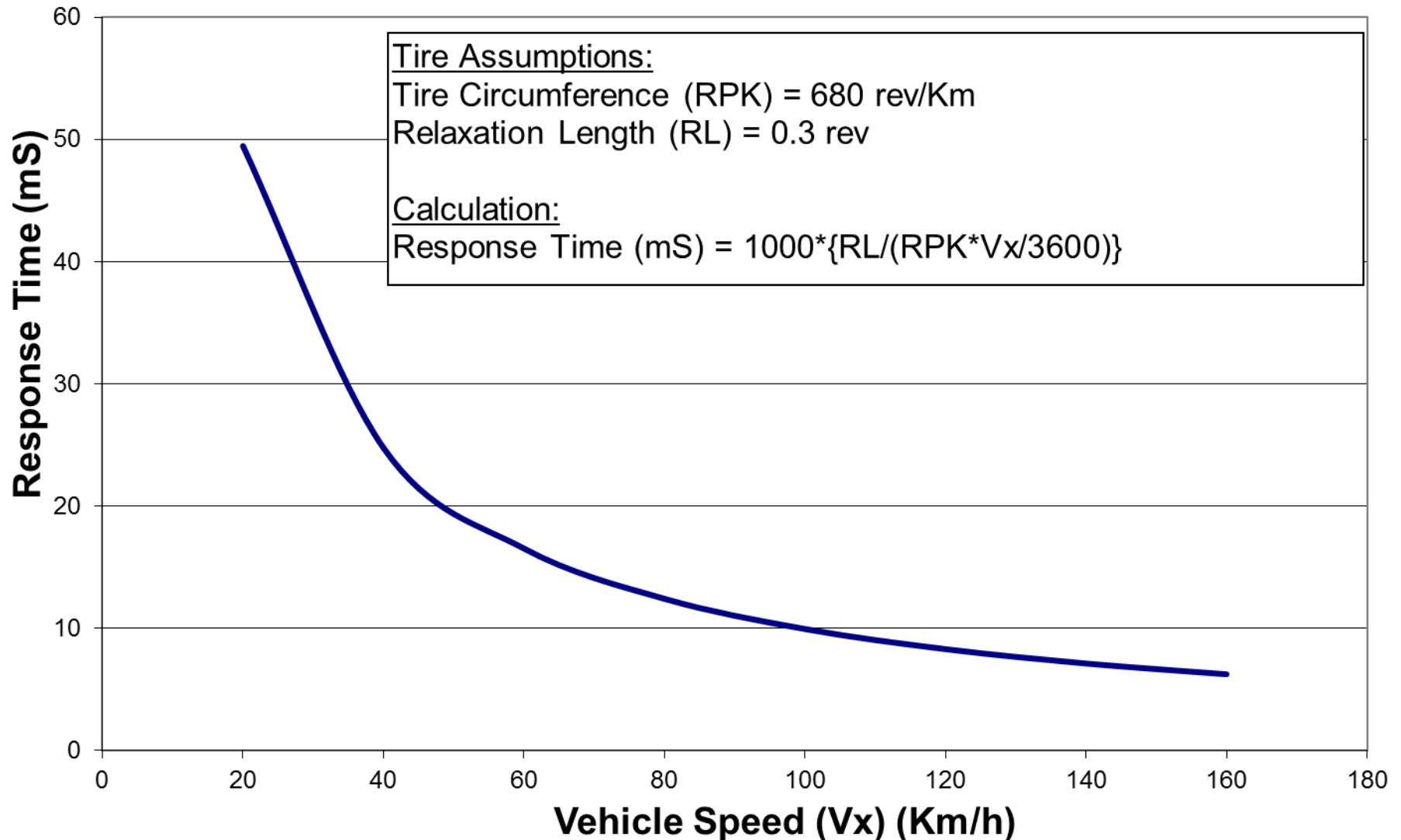
50-90%
rise



- Typically expressed as a fraction of a revolution of the tire
 - Typical range of values for passenger car/light truck tires is 0.2 to 0.4 revolutions
- Affected by tire construction and compound
- Affected by vertical load on the tire

Lateral Force

Lateral Force Response Time versus Vehicle Speed for a Steered Tire

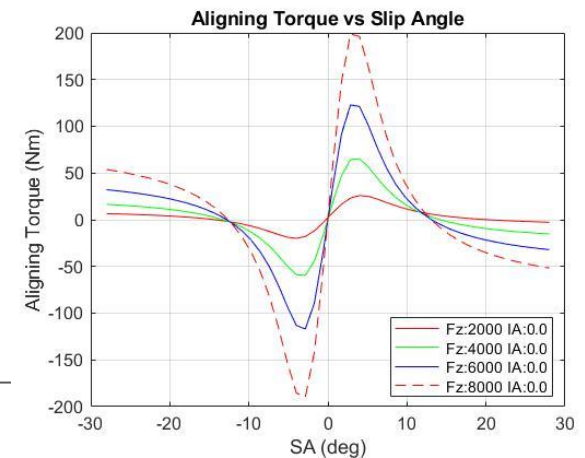
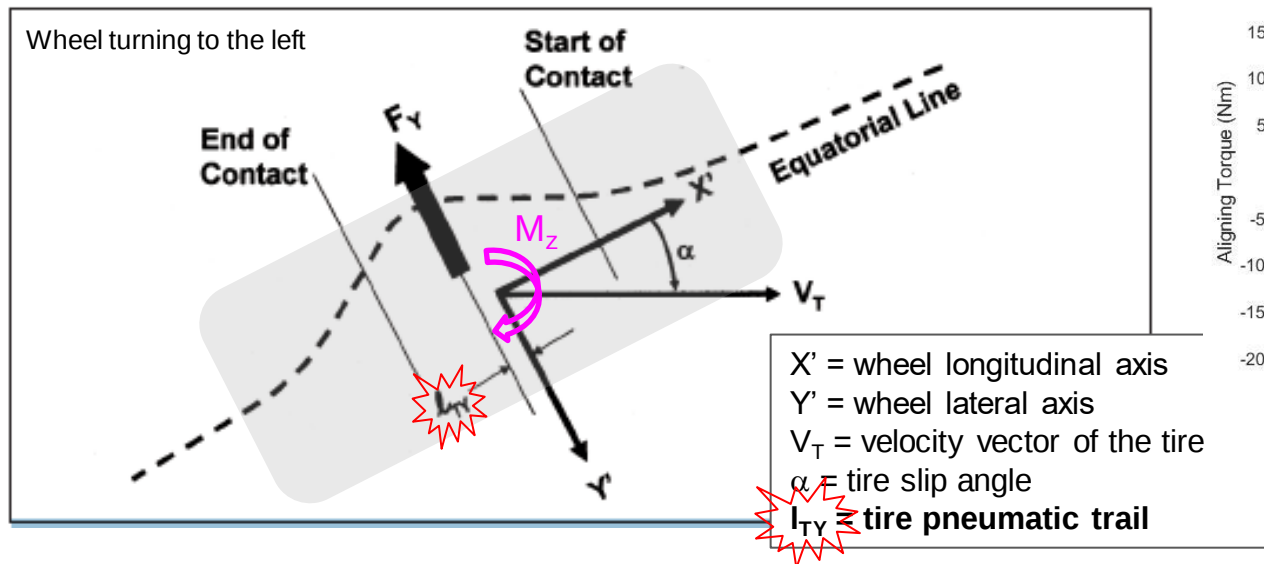


Lateral Force

Pneumatic trail

- The lateral force distribution in the tire contact patch is non-uniform
- The center of lateral force, for a free rolling tire (no longitudinal slip) is aft of the geometric center of the tire a distance called the '**pneumatic trail**'

Figure 8.3: String tire at a slip angle in contact with the road

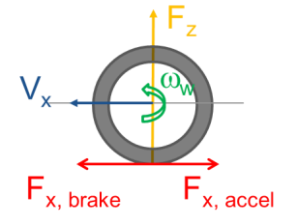


Exemplar data from a tire tested on Flat-Trac[®] machine (surface = 3Mite80 Clearback)

- The lateral force times the pneumatic trail creates an aligning torque, or more precisely, the tire **self-aligning torque**, M_z
 - This is one of the components in the vehicle system that causes a buildup in steering wheel torque when beginning a turn, and the return of the vehicle to straight ahead at the end of a turn

Longitudinal Force

Longitudinal Slip Ratio (as defined in SAE J670e)



7.6 Wheel Spin and Tire Slip

7.6.1 Wheel-Spin Velocity ω_w —The angular velocity of the *wheel* about the Y_w axis.

7.6.2 Reference Wheel-Spin Velocity ω_{w0} —The *wheel-spin velocity* of the *straight free-rolling tire* at a given set of operating conditions.

7.6.3 Tire Longitudinal Slip Velocity—The difference between the *wheel-spin velocity* and the *reference wheel-spin velocity*.

7.6.4 Tire Longitudinal Slip Ratio S_x —The ratio of *tire longitudinal slip velocity* to the *reference wheel-spin velocity*.

$$S_x = \frac{\omega_w - \omega_{w0}}{\omega_{w0}}$$

Longitudinal Force

Longitudinal Slip Ratio (continued)

If angular velocity is to be in rad/s, the **reference wheel-spin velocity** at zero slip angle :

$$\omega_{wo} = \frac{V_x}{R_e}$$

Where:

V_x = Forward speed of the wheel (m/s)
 R_e = Tire effective radius (m)

If angular velocity is to be in rad/s, the **reference wheel-spin velocity** at zero slip angle :

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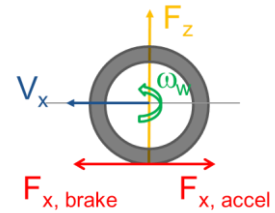
Where:

V_x = Forward speed of the wheel (m/s)
 R_e = Tire effective radius (m)

Longitudinal Force

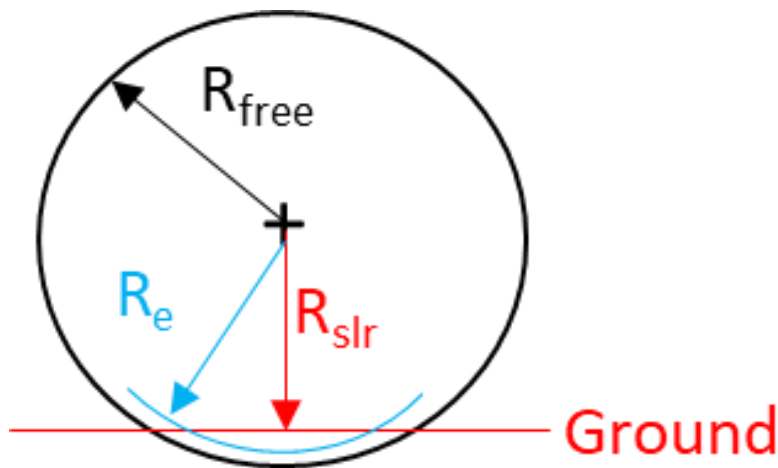
Longitudinal Slip Ratio (continued)

The **tire effective radius, R_e** , is determined experimentally from a F&M test of the tire. Its value is dependent on tire construction, pressure, vertical load and speed of travel.



$$R_{free} > R_e > R_{slr}$$

R_{free} = tire free rolling radius
 R_{slr} = tire static loaded radius



In the absence of a measured value, assume $R_e = R_{slr}$

Longitudinal Force

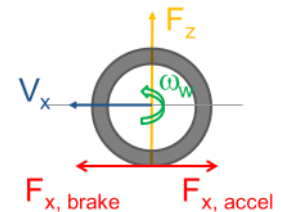
Longitudinal Slip Ratio (continued)

Replacing V_x/R_e for ω_0 in the slip ratio equation we get an alternate form

$$S_x = \frac{\omega_w}{\frac{V_x}{R_e}} - 1$$

When $S_x > 0$, the tire is spinning faster than the forward speed would indicate, therefore we have longitudinal slip in the **acceleration** direction

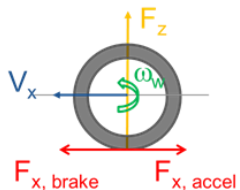
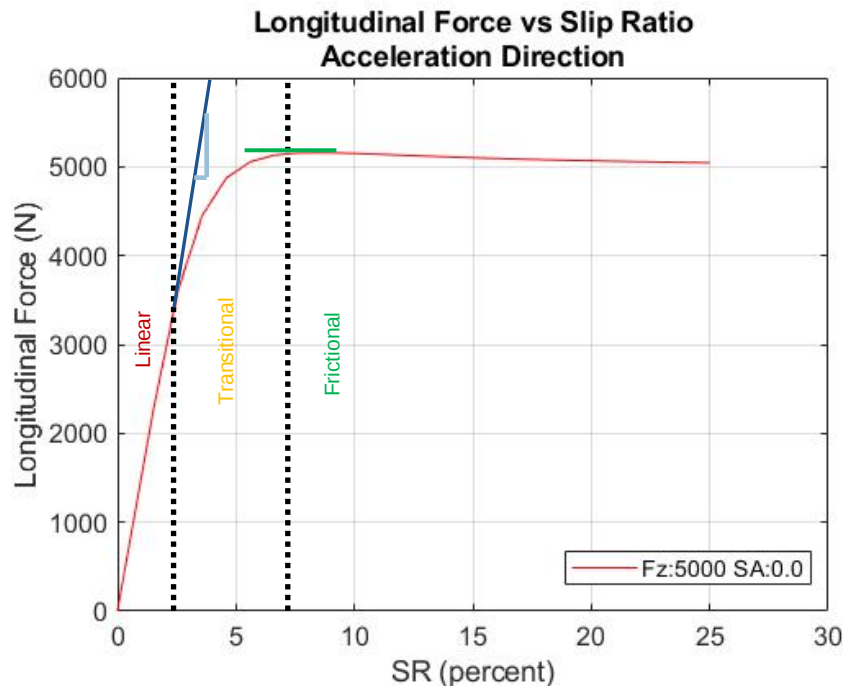
When $S_x < 0$, the tire is spinning slower than the forward speed would indicate, therefore we have longitudinal slip in the **braking** direction



Longitudinal Force

Longitudinal Force (Acceleration shown)

Exemplar data from a tire tested on Flat-Trac[®] machine (surface = 3Mite80 Clearback)



Linear region

- Tire behaves like a linear spring
- **Longitudinal stiffness** defined in this area (i.e. slope at 0% slip ratio)

Transitional region

- Tire is moving from linear spring behavior to frictional behavior
- Longitudinal stiffness decreasing with more slip ratio

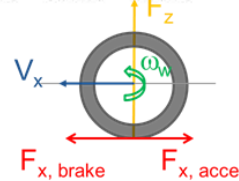
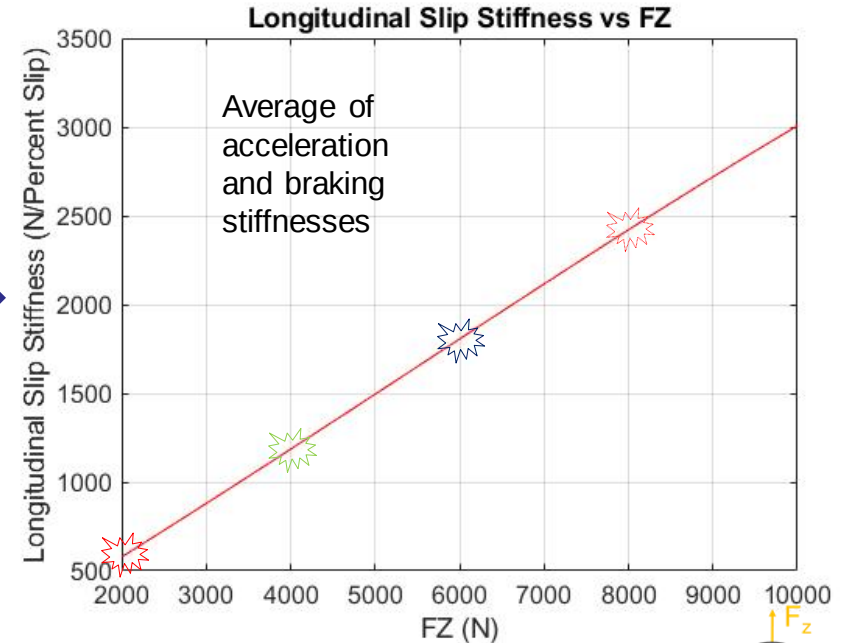
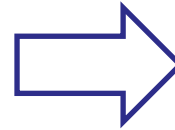
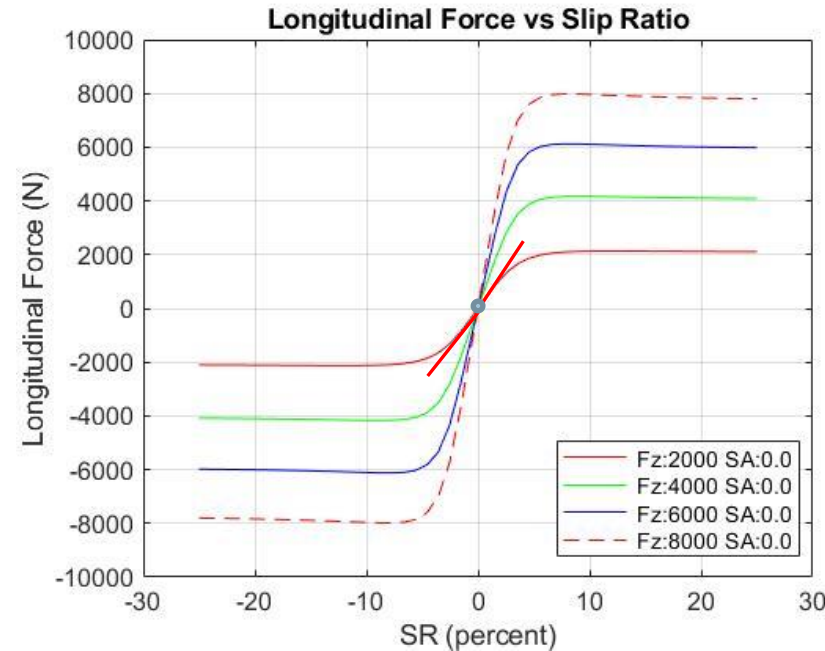
Frictional region

- Tire behaves more like rubber sliding on pavement
- Peak/saturated longitudinal force occur in this area (“***grip***”)

Longitudinal Force

Longitudinal Stiffness (Slope of F_x /Slip Ratio curve at 0% Slip)

Exemplar data from a tire tested on Flat-Trac[®] machine (surface = 3Mite80 Clearback)

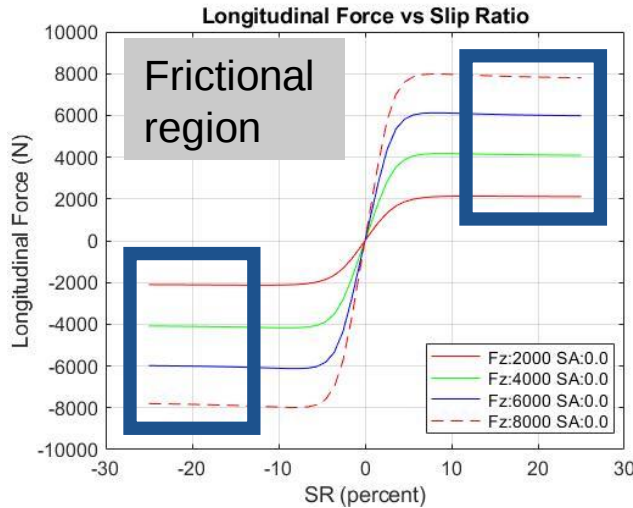
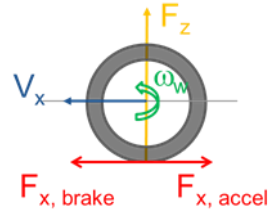


Longitudinal stiffness:

- Changes with vertical load and inclination angle (slightly)
- Characterizes how effective ("stiff") the tire is in generating longitudinal force
- Is a key tire parameter affecting vehicle accel/braking response to a throttle/brake input
- Is affected by compound, tread block design, tire size, tire temperature, etc.

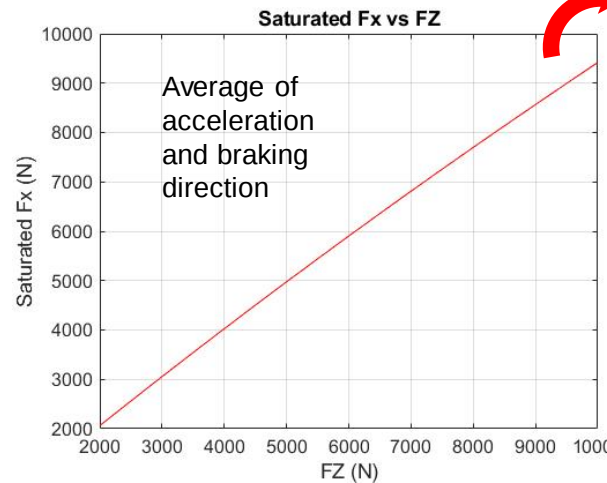
Longitudinal Force

Saturated Longitudinal Force (aka 'grip force')



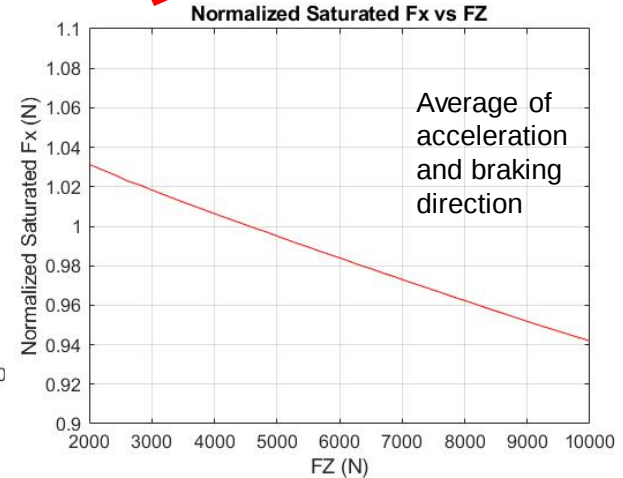
Saturated longitudinal force

- Tire rubber sliding on pavement
- “Saturated” = Increasing slip ratio doesn't generate increased longitudinal force



Variation with Fz

- Saturated Fx increases with increasing Fz



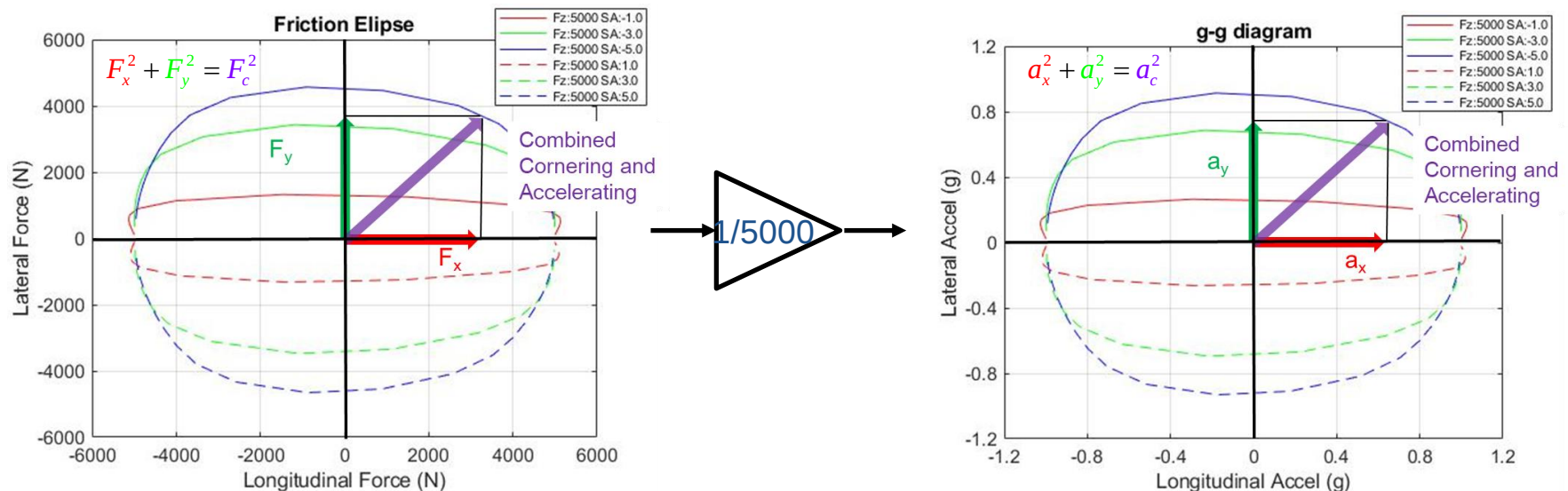
Normalized Sat. Fx – “Mu”

- Mu decreases with increasing Fz

- Measure of how “sticky” a tire is at high slip ratios
- Usually expressed as a dimensionless value - coeff. of friction (μ)
- Key tire parameter that affects max. Ax, limit handling balance, etc.
- Affected by compound, tire size, tire temperature, etc.

Friction Ellipse/g-g Diagram

When we plot the lateral force at a given vertical load and slip angle versus the peak longitudinal force at a given vertical load and slip angle, we get the so-called '**Friction Ellipse**'



Given a desired amount of **lateral** tire force, there is a amount of **longitudinal** tire force available, subject to the constraint that the **vector sum** cannot exceed the outer limit of the friction ellipse

If we divide the lateral and longitudinal force by the corner weight, we get the so-called '**g-g diagram**' which shows the capacity of the tire in terms of planar acceleration

Summary

- All forces (other than aero forces) that affect the motion of the vehicle come through the tires
- Tires have the biggest effect on determining vehicle's ride, steering and handling characteristics
- Tires have a number of conflicting requirements and affect many attributes
- Tire slip angle is the difference between the tire direction of travel and its heading
- Tire longitudinal slip ratio is the ratio of the tire actual rotational speed to its ideal speed based on forward velocity

Summary

- A tire has 3 force generation regions: linear, transitional and frictional
 - In the linear region the tire behaves like a spring
 - In the frictional region, the tire behaves like rubber sliding on pavement
- Tire forces (and moments) are sensitive to speed of rotation, the vertical force on the tire and the inclination angle of the tire
- The amount of available tire lateral force for a given longitudinal force (and vice versa) is limited by the size and shape of the tire friction ellipse

Questions?