Vehicle Primary Ride Dynamics
Part 1: Modelling and Analysis

Tim Drotar (TD)

About the author

Tim Drotar is currently a lead engineer in advanced vehicle dynamics at Stellantis. Previously, he spent 30 years at Ford Motor Company where he specialized in chassis systems and vehicle dynamics for passenger cars and light trucks. Tim is a member of SAE, SCCA and The Tire Society. He holds a B.S. in Mechanical Engineering from Lawrence Technological University and a M.S. in Mechanical Engineering from the University of Michigan-Dearborn.

Tim also teaches the following classes for SAE:

- Advanced Vehicle Dynamics for Passenger Cars and Light Trucks
  - https://www.sae.org/learn/content/c0415/
- Fundamentals of Steering Systems
  - https://www.sae.org/learn/content/c0716/

Outline

- Learning Objectives
- References
- Introduction to Primary Ride
- The 4-DOF Primary Ride Model
- The 2-DOF Primary Ride Model
- Maurice Olley and Guidelines for Primary Ride
- Summary

Tim Drotar/tdrotar08@gmail.com

Learning Objectives

At the end of this presentation, you should be able to:

- Explain what primary ride is
- Calculate vertical road disturbances as a function of road and vehicle parameters
- Identify the parameters associated with the 4-dof primary ride model
- Use 4-dof model to estimate body-on-chassis sprung mass natural and wheel hop and frequencies
- Identify the parameters associated with the 2-dof primary ride model
- Use the 2-dof model to estimate the body pitch and bounce natural frequencies
- Locate the body pitch and bounce nodes
- Compare the results with general guidelines

References

These notes were created from several sources, including:

Mola, Simone. Chassis Design – ME421 Class notes. Flint MI: General Motors Institute, 1991.

Milliken, William and Milliken, Douglas, Chassis Design: Principles and Analysis. Warrendale, PA: Society of Automotive Engineers, 2002.

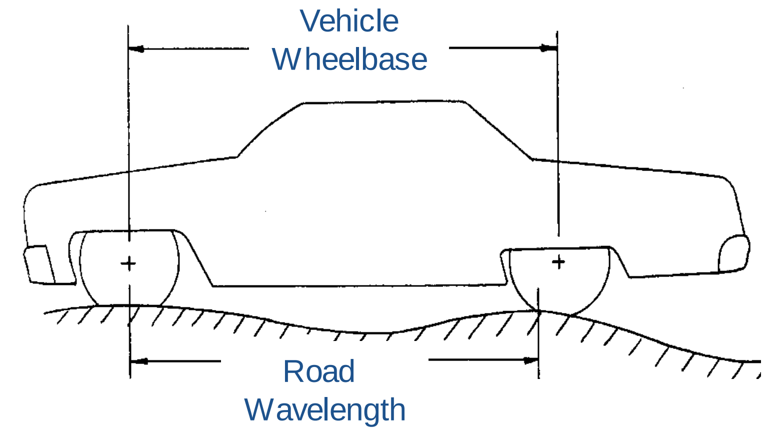
Gillespie, Thomas. Fundamentals of Vehicle Dynamics. Warrendale, PA: Society of Automotive Engineers, 1992.

Olley, Maurice. Independent Wheel Suspension – Its Whys and Wherefores. Detroit, MI: Society of Automotive Engineers, 1934

What is the ride attribute?

Characterize the vehicle body and occupant displacement, velocity and acceleration while driving on different road surfaces

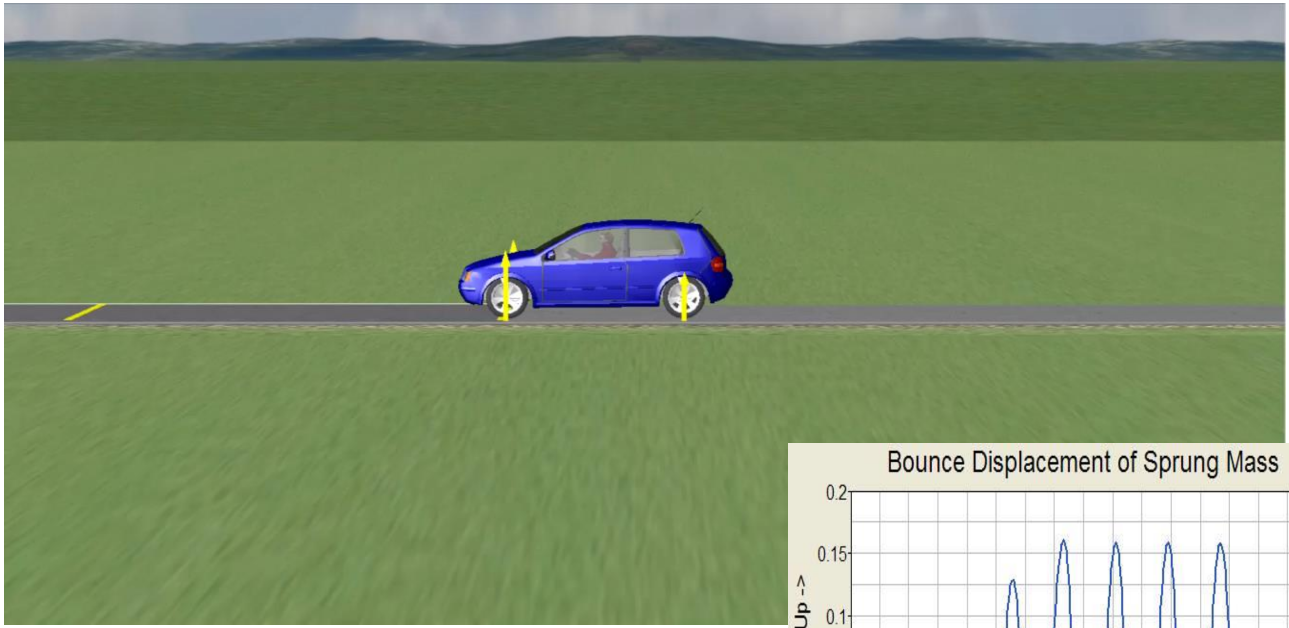
Primary Ride pertains to the rigid body motion of the vehicle sprung mass. Motions considered to be primary ride are bounce, pitch and roll. The frequency range of interest is quite low (approx. 0-5 Hz) and the displacements are relatively large, on the order of *inches*.



Secondary Ride pertains to higher frequency and smaller amplitude displacement of the body and chassis. The frequency range of interest is approx. 5-100 Hz for tactile and above 500 Hz for audible. Displacements are relatively small, on the order of *millimeters*

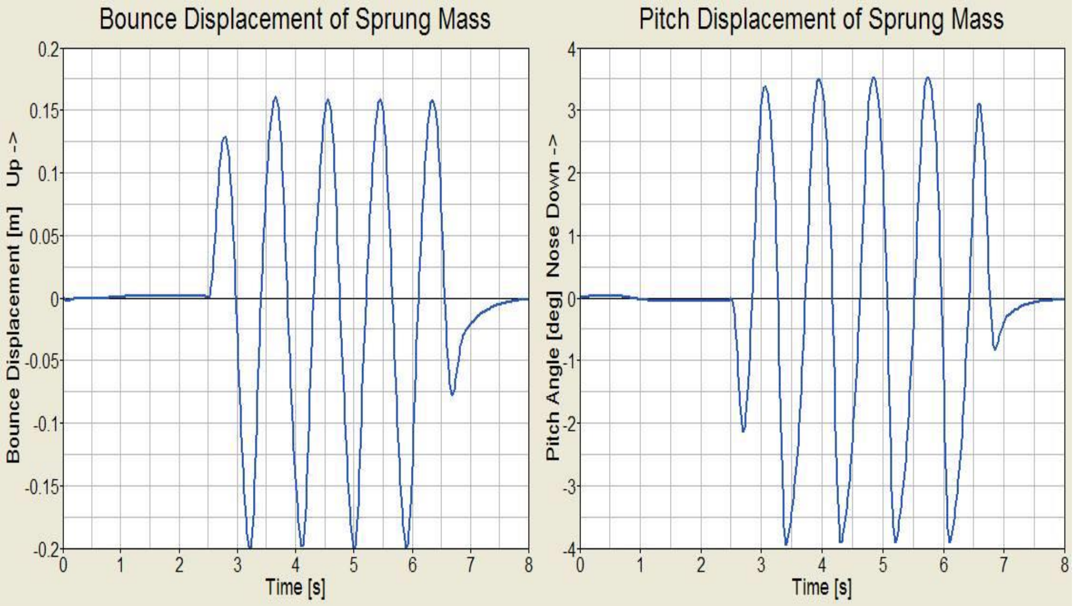
Belgium Block Road – Ford Lommel Proving Grounds

CarSim primary ride simulation of C-class hatchback

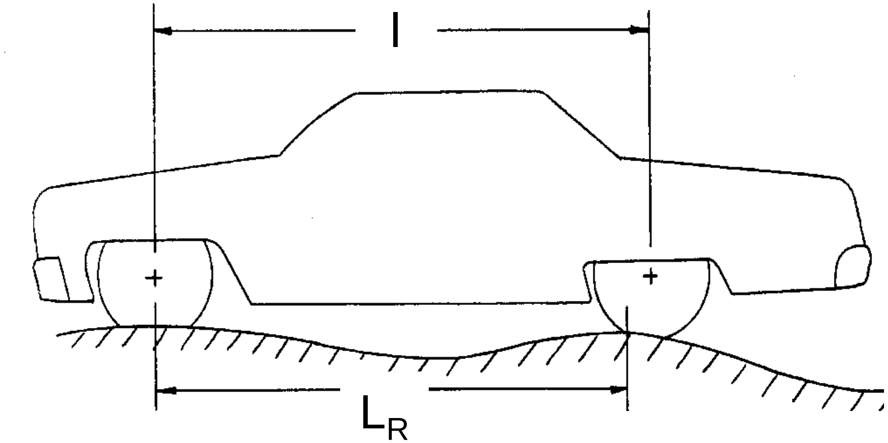


<- Simulation

Sprung mass response ->



Primary Ride Disturbances – Bounce and Pitch



l (m) = Vehicle wheelbase

V_v (kph) = Vehicle velocity

L_R (m) = Road roughness spatial wavelength

$0.600 < L_R(m) < 60.0$ typ.

A_R (m) = Road roughness

$0.025 < A_R(m) < 100$ typ.

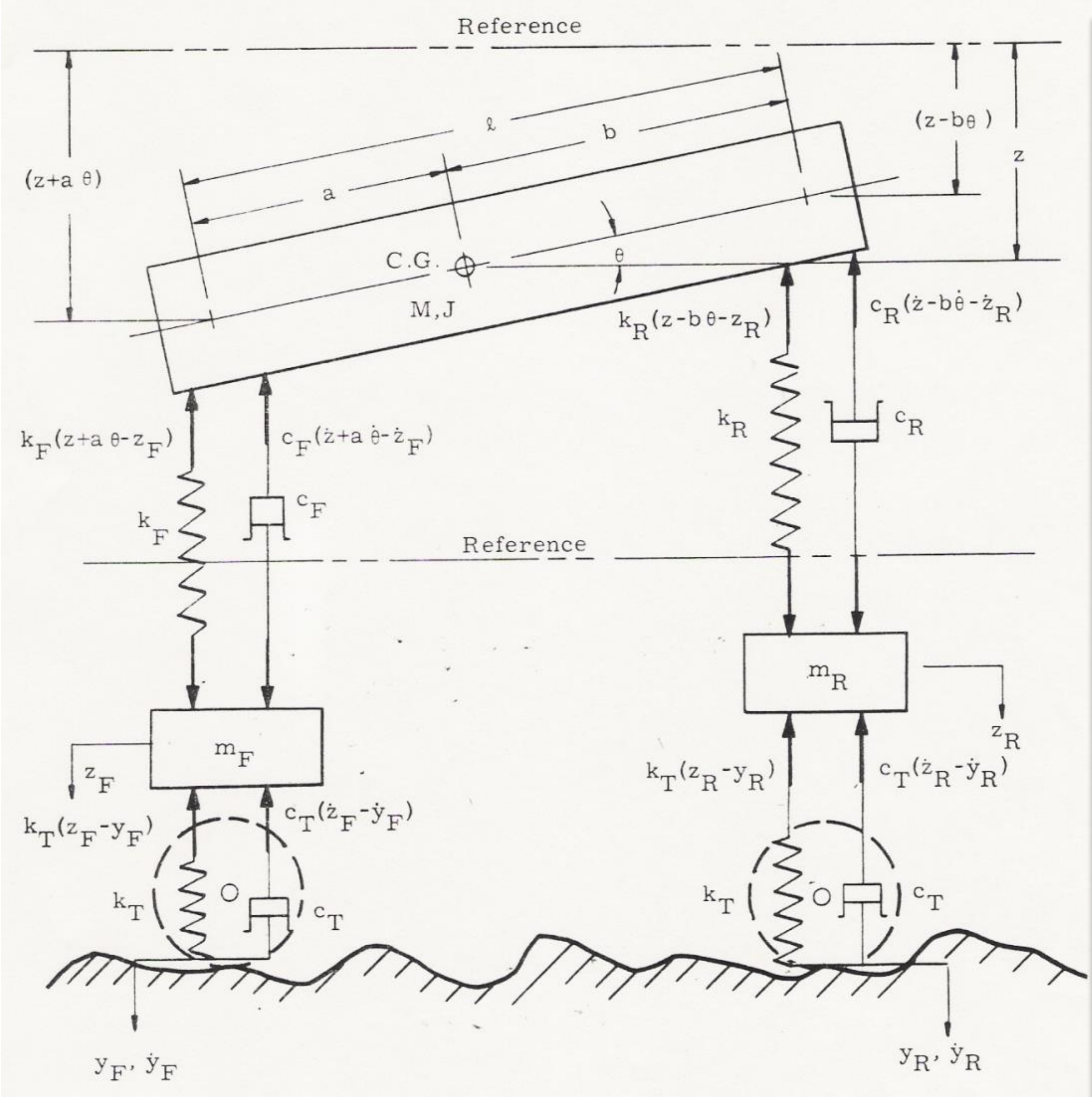
t (s) = time

Road roughness input frequency (Hz) $$f_R = \frac{1}{2\pi}\left[\frac{1}{3.6}\frac{V_v}{L_R}\right]$$

Road input displacement (m) $$y_R = A_R \sin[(2\pi f_R)t] = A_R \sin\left[\left(\frac{1}{3.6}\frac{V_v}{L_R}\right)t\right]$$

Road input velocity (m/s) $$\frac{dy_R}{dt} = 2\pi f_R A_R \cos[(2\pi f_R)t] = \frac{1}{3.6}\frac{V_V A_R}{L_R}\cos\left[\left(\frac{1}{3.6}\frac{V_V}{L_R}\right)t\right]$$

The 4-DOF Primary Ride Model



(1) The terms 'spring rate' and 'spring stiffness' are synonymous

M (kg) = Sprung mass

J (kg-m^2) = Sprung mass inertia

a, b (m) = Distance from sprung mass Cg to front, rear axle

l (m) = Wheelbase

θ (rad) = Pitch angle

z (m) = Vertical displacement of sprung mass

k_F, k_R (N/m) = 2x front, rear suspension rate[1] at the wheel

c_F, c_R (N-s/m) = 2x front, rear suspension damping at the wheel

m_F, m_R (kg) = Unsprung mass

z_F, z_R (m) = Vertical Displacement of unsprung mass

k_T (N/m) = 2x tire rate[1]

c_T (N-s/m) = 2x tire damping

y_F, y_R (m) = Vertical road input displacement

$\dot{y}_F, \dot{y}_R$ (m/s) = Vertical road input velocity

Applying Newton's Law of Motion to Sprung & Unsprung Mass:

Vertical Motion (bounce) of Sprung Mass (body)

$$\Sigma F_Z = M\ddot{Z}$$

$$-k_F(z + a\theta - z_F) - c_F(\dot{z} + a\dot{\theta} - \dot{z}_F) - k_R(z - b\theta - z_R) - c_R(\dot{z} - b\dot{\theta} - \dot{z}_R) = M\ddot{Z}$$

Rotational Motion (pitch) of Sprung Mass (body)

$$\Sigma T_M = J\ddot{\theta}$$

$$-k_F a(z + a\theta - z_F) - c_F a(\dot{z} + a\dot{\theta} - \dot{z}_F) - k_R b(z - b\theta - z_R) - c_R b(\dot{z} - b\dot{\theta} - \dot{z}_R) = J\ddot{\theta}$$

Vertical Motion (wheel hop) of Front Axle Unsprung Mass

$$\Sigma F_Z = m_F\ddot{Z}_F$$

$$-k_T(z_F - y_F) + c_F(\dot{z} + a\dot{\theta} - \dot{z}_F) - k_F(z + a\theta - z_F) - c_T\left(\dot{z}_F - \dot{y}_F\right) = m_F\ddot{Z}_F$$

Vertical Motion (wheel hop) of Rear Axle Unsprung Mass

$$\Sigma F_Z = m_R\ddot{Z}_R$$

$$-k_T(z_R - y_R) + c_R(\dot{z} - b\dot{\theta} - \dot{z}_R) - k_R(z - b\theta - z_R) - c_T\left(\dot{z}_R - \dot{y}_R\right) = m_R\ddot{Z}_R$$

Determining the sprung mass bounce natural frequency

Vertical Motion (bounce) of Sprung Mass (body)

$$\Sigma F_Z = M\ddot{z}$$

$$-k_F(z + a\theta - z_F) - c_F(\dot{z} + a\dot{\theta} - \dot{z}_F) - k_R(z - b\theta - z_R) - c_R(\dot{z} - b\dot{\theta} - \dot{z}_R) = M\ddot{z}$$

Rearrange to Obtain Classical Mechanical Vibration Format

$$\ddot{z} + \left(\frac{c_F + c_R}{M}\right)\dot{z} + \left(\frac{k_F + k_R}{M}\right)z + \left(\frac{ac_F - bc_R}{M}\right)\dot{\theta} + \left(\frac{ak_F - bk_R}{M}\right)\theta - \left(\frac{c_F}{M}\right)\dot{z}_F$$
$$-\left(\frac{k_F}{M}\right)z_F - \left(\frac{c_R}{M}\right)\dot{z}_R + \left(\frac{k_R}{M}\right)z_R = 0$$

Sprung Mass (Body) Bounce Natural Frequency

$$\omega_{B,O}\left(\frac{rad}{\sec}\right) = \sqrt{\frac{k_F + k_R}{M}}$$

$$f_{B,O}(Hz) = \frac{1}{2\pi}\sqrt{\frac{k_F + k_R}{M}}$$

Sprung Mass Bounce Natural Frequency (Body on Chassis)

Determining the sprung mass pitch natural frequency

Rotational Motion (pitch) of Sprung Mass (body)

$$\Sigma T_M = J\ddot{\theta}$$

$$-k_F a(z + a\theta - z_F) - c_F a(\dot{z} + a\dot{\theta} - \dot{z}_F) - k_R b(z - b\theta - z_R) - c_R b(\dot{z} - b\dot{\theta} - \dot{z}_R) = J\ddot{\theta}$$

Rearrange to Obtain Classical Mechanical Vibration Format

$$\ddot{\theta} + \left(\frac{a^2 c_F + b^2 c_R}{J}\right)\dot{\theta} + \left(\frac{a^2 k_F + b^2 k_R}{J}\right)\theta + \left(\frac{ac_F - bc_R}{J}\right)\dot{z} + \left(\frac{ak_F - bk_R}{J}\right)z - \left(\frac{ac_F}{J}\right)\dot{z}_F$$

$$-\left(\frac{ak_F}{J}\right)z_F - \left(\frac{bc_R}{J}\right)\dot{z}_R + \left(\frac{bk_R}{J}\right)z_R = 0$$

Sprung Mass (Body) Pitch Natural Frequency

$$\omega_{P,O}\left(\frac{rad}{\sec}\right) = \sqrt{\frac{a^2 k_F + b^2 k_R}{J}}$$

$$f_{P,O}(Hz) = \frac{1}{2\pi}\sqrt{\frac{a^2 k_F + b^2 k_R}{J}}$$

Sprung Mass Pitch Natural Frequency (Body on Chassis)

Determining the front axle wheel hop natural frequency

Vertical Motion (wheel hop) of Front Axle Unsprung Mass

$$\Sigma F_Z = m_F \ddot{Z}_F$$

$$-k_T(z_F - y_F) + c_F(\dot{z} + a\dot{\theta} - \dot{z}_F) - k_F(z + a\theta - z_F) - c_T\left(\dot{z}_F - \dot{y}_F\right) = m_F \ddot{Z}_F$$

Rearrange to Obtain Classical Mechanical Vibration Format

$$\ddot{z}_F + \left(\frac{c_F + c_T}{m_F}\right)\dot{z}_F + \left(\frac{k_F + k_T}{m_F}\right)z_F - \left(\frac{c_F}{m_F}\right)\dot{z} - \left(\frac{k_F}{m_F}\right)z - \left(\frac{ac_F}{m_F}\right)\dot{\theta} - \left(\frac{ak_F}{m_F}\right)\theta = \left(\frac{c_T}{m_F}\right)\dot{y}_F + \left(\frac{k_T}{m_F}\right)y_F$$

Front Axle Unsprung Mass (wheel hop) Natural Frequency

$$\omega_{HF}\left(\frac{rad}{\sec}\right) = \sqrt{\frac{k_F + k_T}{m_F}}$$

$$f_{HF}(Hz) = \frac{1}{2\pi}\sqrt{\frac{k_F + k_T}{m_F}}$$

Front Axle Wheel Hop Natural Frequency

Determining the rear axle wheel hop natural frequency

Vertical Motion (wheel hop) of Rear Axle Unsprung Mass

$$\Sigma F_Z = m_R \ddot{Z}_R$$

$$-k_T(z_R - y_R) + c_R\left(\dot{z} + a\dot{\theta} - \dot{z}_R\right) - k_R(z + a\theta - z_R) - c_T\left(\dot{z}_R - \dot{y}_R\right) = m_R \ddot{Z}_R$$

Rearrange to Obtain Classical Mechanical Vibration Format

$$\ddot{z}_R + \left(\frac{c_R + c_T}{m_R}\right)\dot{z}_R + \left(\frac{k_R + k_T}{m_R}\right)z_R - \left(\frac{c_R}{m_R}\right)\dot{z} - \left(\frac{k_R}{m_R}\right)z - \left(\frac{ac_R}{m_R}\right)\dot{\theta} - \left(\frac{ak_R}{m_R}\right)\theta = \left(\frac{c_T}{m_R}\right)\dot{y}_R + \left(\frac{k_T}{m_R}\right)y_R$$

Rear Axle Unsprung Mass (wheel hop) Natural Frequency

$$\omega_{HR}\left(\frac{rad}{\sec}\right) = \sqrt{\frac{k_R + k_T}{m_R}}$$

$$f_{HR}(Hz) = \frac{1}{2\pi}\sqrt{\frac{k_R + k_T}{m_R}}$$

Rear Axle Wheel Hop Natural Frequency

Exemplar values for wheel hop natural frequencies

input

calculated

2019 Jeep Cherokee Trailhawk AWD @ SSF loading condition

$$f_{HF}(Hz) = \frac{1}{2\pi}\sqrt{\frac{k_F + k_T}{m_F}}$$

$$f_{HR}(Hz) = \frac{1}{2\pi}\sqrt{\frac{k_R + k_T}{m_R}}$$

Inputs

a	mm	1169.6	cg to frt axle
b	mm	1552.2	cg to rear axle
M_V	kg	2083.0	mass
$M_{V,F}$	kg	1187.9	front mass
$M_{V,R}$	kg	895.1	rear mass
m_F	kg	118.8	front unsprung mass (assume 10% of total front mass)
m_R	kg	89.5	rear unsprung mass (assume 10% of total rear mass)
M_F	kg	1069.1	front sprung mass (assume 10% unsprung)
M_R	kg	805.6	rear sprung mass (assume 10% unsprung)
M	kg	1874.7	Sprung mass
K_F	N/mm	73.9	2x front suspension rate
K_R	N/mm	81.0	2x rear suspension rate
$K_{T,F}$	N/mm	600.0	2x tire rate
$K_{T,R}$	N/mm	600.0	2x tire rate
K'_F	N/mm	65.8	LF + RF ride rate
K'_R	N/mm	71.3	LR + RR ride rate
l	kgm^2	2721.8	wheelbase
J_V	kgm^2	3526.0	Total vehicle inertia
J	kgm^2	3147.8	Sprung mass pitch inertia

Vehicle parameters

Outputs

Wheel hop natural frequencies

f_{HF}	11.99	hz
f_{HR}	13.88	hz

Wheel hop natural frequencies

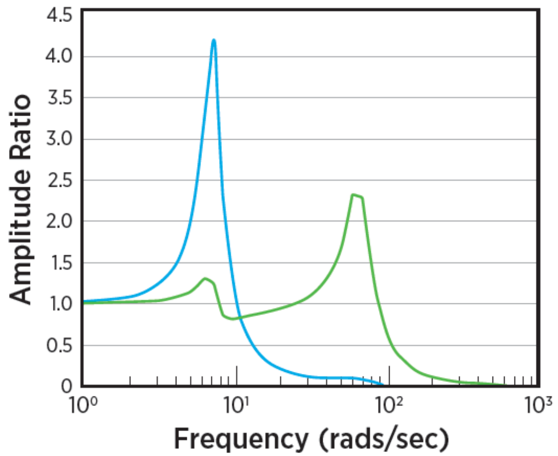
Microsoft Excel Worksheet

Tim Drotar/tdrotar08@gmail.com

The 4-DOF Primary Ride Model

Some comments about the 4-DOF Ride Model

- Can be used to assess the forced pitch and bounce response of the sprung mass
- Can be used to assess the transmissibility between the unsprung mass response and the sprung mass response



Sprung Mass Bounce
Unsprung Mass Vertical

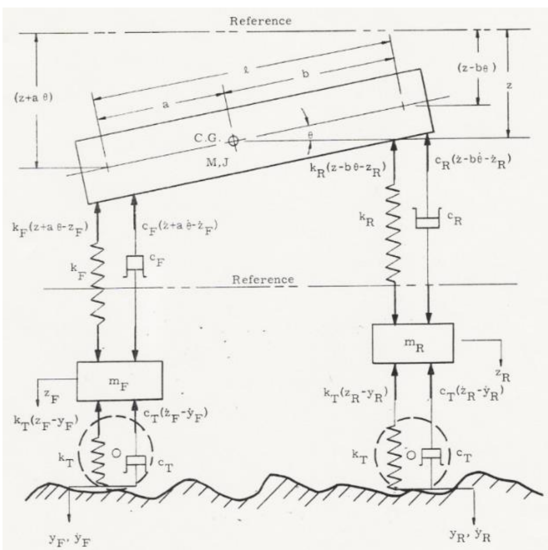
- Switching from an ‘axle model’ (left and right lumped together) to a ‘4-corner model’ allows for additional analysis of the roll degree of freedom (aka 7-dof model)

The 2-DOF Primary Ride Model

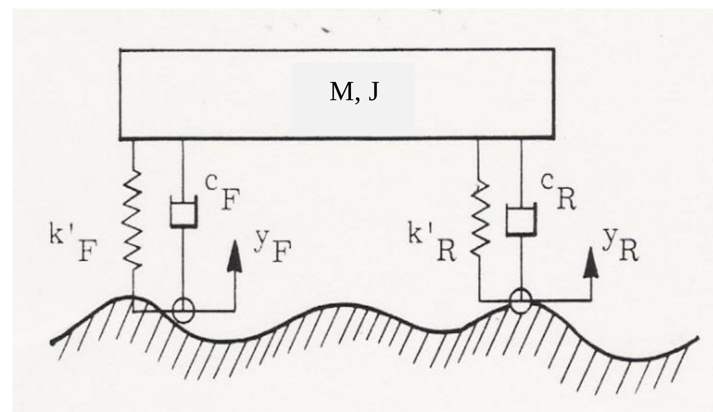
If we make the following simplifications:

1. Treat the suspension vertical stiffness at the wheel and the tire vertical stiffness as springs in series
2. Assume tire damping is much less than suspension damping

We can reduce the 4-dof primary ride model to a 2-dof model and directly get the sprung mass response as a function of road input



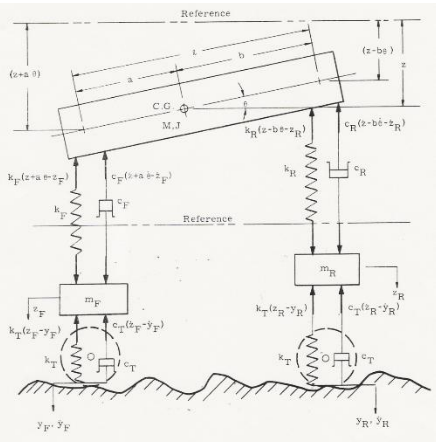
(4 coupled equations)

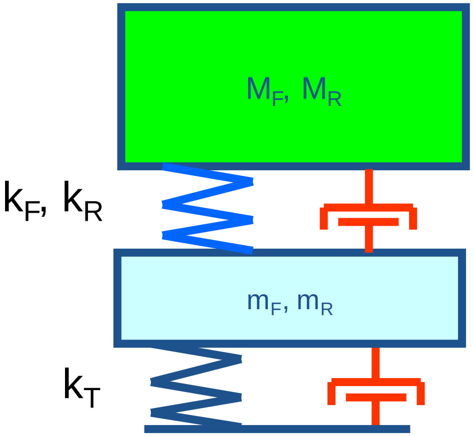


(2 coupled equations)

2-DOF Primary Ride Model

Taking the suspension rates[1] k_F, k_R in series with the tire rate, k_T, give us the ride rates, k'_F, k'_R

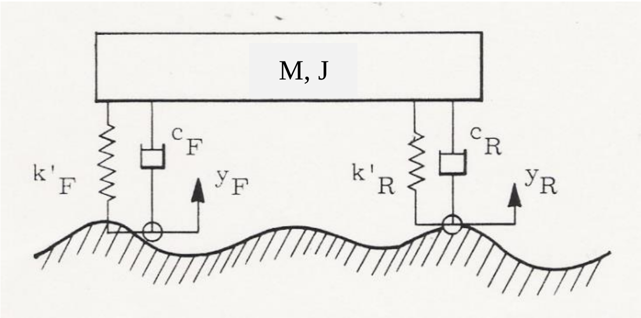




$$\frac{1}{k'_F} = \frac{1}{k_F} + \frac{1}{k_T}$$

$$k'_F = \frac{k_F k_T}{k_F + k_T}$$

$$k'_R = \frac{k_R k_T}{k_R + k_T}$$



M (kg) = Sprung mass

J (kg-m^2) = Sprung mass inertia

a, b (m) = Distance from sprung mass Cg to front, rear axle

l (m) = Wheelbase

θ (rad) = Pitch angle of sprung mass

z (m) = Vertical displacement of sprung mass

k'_F, k'_R (N/m) = 2x front, rear ride rate

c_F, c_R (N-s/m) = 2x front, rear suspension damping at the wheel

m_F, m_R (kg) = Unsprung mass

y_F, y_R (m) = Vertical road input displacement

(1) The terms ‘spring rate’ and ‘spring stiffness’ are synonymous

Ride frequency and frequency ratio

Knowing the axle ride rate (2x corner) and the axle sprung mass, we can calculate the sprung mass natural frequency of the front and rear of the vehicle. These are known as the ‘ride frequencies.

$$f_{R,F} = \frac{1}{2\pi}\sqrt{\frac{k'_F}{m_F}}$$

Front Ride Natural Frequency (Hz)

$$f_{R,R} = \frac{1}{2\pi}\sqrt{\frac{k'_R}{m_R}}$$

Rear Ride Natural Frequency (Hz)

The ratio of rear-to-front ride frequencies is called the **Ride Frequency Ratio**

$$R_R = \frac{f_{R,r}}{f_{R,f}}$$

Ride Frequency Ratio (-)

We will see later that there are ‘guidelines’ for these values that will enable good primary ride in the absence of damping

Tim Drotar/tdrotar08@gmail.com

Exemplar values for ride frequency and frequency ratio

Microsoft Excel Worksheet

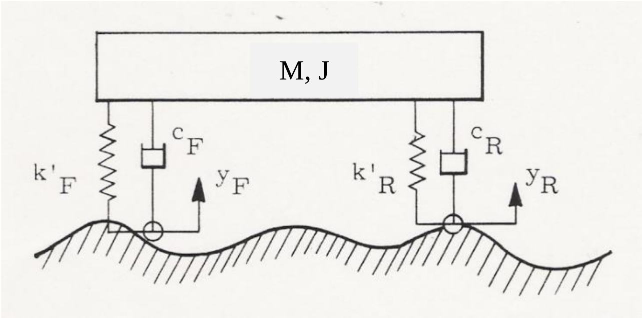
(green)	input
(yellow)	calculated

2019 Jeep Cherokee Trailhawk AWD @ SSF loading condition

Inputs

a	mm	1169.6	cg to frt axle
b	mm	1552.2	cg to rear axle
M_V	kg	2083.0	mass
$M_{V,F}$	kg	1187.9	front mass
$M_{V,R}$	kg	895.1	rear mass
m_F	kg	118.8	front unsprung mass (assume 10% of total front mass)
m_R	kg	89.5	rear unsprung mass (assume 10% of total rear mass)
M_F	kg	1069.1	front sprung mass (assume 10% unsprung)
M_R	kg	805.6	rear sprung mass (assume 10% unsprung)
M	kg	1874.7	Sprung mass
K_F	N/mm	73.9	2x front suspension rate
K_R	N/mm	81.0	2x rear suspension rate
$K_{T,F}$	N/mm	600.0	2x tire rate
$K_{T,R}$	N/mm	600.0	2x tire rate
K'_F	N/mm	65.8	LF + RF ride rate
K'_R	N/mm	71.3	LR + RR ride rate
l	kgm^2	2721.8	wheelbase
J_V	kgm^2	3526.0	Total vehicle inertia
J	kgm^2	3147.8	Sprung mass pitch inertia

Vehicle parameters



Outputs

Wheel hop natural frequencies

f_{HF}	11.99	hz
f_{HR}	13.88	hz

Wheel hop natural frequencies

Ride Frequencies

f_{RF}	1.25	hz
f_{RR}	1.50	hz
R_R	1.20	(f_{RR}/f_{RF})

Ride frequencies and frequency ratio

Tim Drotar/tdrotar08@gmail.com

The 2-DOF Primary Ride Model

<u>Sprung mass bounce and pitch in classical mechanical vibration format</u>

Vertical Motion (Bounce) of Sprung Mass

$$\ddot{z} + \left(\frac{c_F + c_R}{M}\right)\dot{z} + \left(\frac{k'_F + k'_R}{M}\right)z + \left(\frac{bc_R - ac_F}{M}\right)\dot{\theta} + \left(\frac{bk'_F - ak'_F}{M}\right)\theta$$
$$= \left(\frac{c_F}{M}\right)\dot{y}_F + \left(\frac{k'_F}{M}\right)y_F + \left(\frac{c_R}{M}\right)\dot{y}_R + \left(\frac{k'_R}{M}\right)y_R$$

Rotational Motion (Pitch) of Sprung Mass

$$\ddot{\theta} + \left(\frac{a^2c_F + b^2c_R}{J}\right)\dot{\theta} + \left(\frac{a^2k'_F + b^2k'_R}{J}\right)\theta + \left(\frac{bc_R - ac_F}{J}\right)\dot{z} + \left(\frac{bk'_R - ak'_F}{J}\right)z$$
$$= \left(\frac{ac_F}{J}\right)\dot{y}_F + \left(\frac{ak'_F}{J}\right)y_F - \left(\frac{bc_R}{J}\right)\dot{y}_R - \left(\frac{bk'_R}{J}\right)y_R$$

Sprung mass bounce and pitch transfer functions in classical mechanical vibration format

Vertical Motion (Bounce) of Sprung Mass

$$TF_z(s) = \frac{z(s)}{y_R(s)}$$

$$\ddot{z} + \left(\frac{c_F + c_R}{M}\right)\dot{z} + \left(\frac{k'_F + k'_R}{M}\right)z + \left(\frac{bc_R - ac_F}{M}\right)\dot{\theta} + \left(\frac{bk'_F - ak'_F}{M}\right)\theta =$$
$$\left(\frac{c_F}{M}\right)\dot{y}_F + \left(\frac{k'_F}{M}\right)y_F + \left(\frac{c_R}{M}\right)\dot{y}_R + \left(\frac{k'_R}{M}\right)y_R$$

Rotational Motion (Pitch) of Sprung Mass

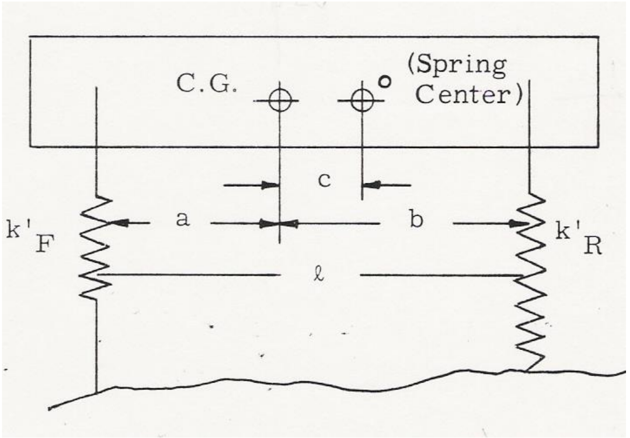
$$TF_\theta(s) = \frac{\theta(s)}{y_R(s)}$$

$$\ddot{\theta} + \left(\frac{a^2c_F + b^2c_R}{J}\right)\dot{\theta} + \left(\frac{a^2k'_F + b^2k'_R}{J}\right)\theta + \left(\frac{bc_R - ac_F}{J}\right)\dot{z} + \left(\frac{bk'_R - ak'_F}{J}\right)z =$$
$$\left(\frac{ac'_F}{J}\right)\dot{y}_F + \left(\frac{ak'_F}{J}\right)y_F - \left(\frac{bc_R}{J}\right)\dot{y}_R - \left(\frac{bk_R}{J}\right)y_R$$

Two Degree of Freedom Undamped Primary Ride Model

Vertical Motion (Bounce) of Sprung Mass Without Damping

$$\ddot{z} + \left(\frac{k'_F + k'_R}{M}\right)z + \left(\frac{bk'_R - ak'_F}{M}\right)\theta = 0$$



Rotational Motion (Pitch) of Sprung Mass Without Damping

$$\ddot{\theta} + \left(\frac{a^2k'_F + b^2k'_R}{J}\right)\theta + \left(\frac{bk'_R - ak'_F}{J}\right)z = 0$$

Defining the following coefficients and substituting into the above equations:

$$\alpha = \left(\frac{k'_F + k'_R}{M}\right)$$

$$\beta = \left(\frac{bk'_R - ak'_F}{M}\right)$$

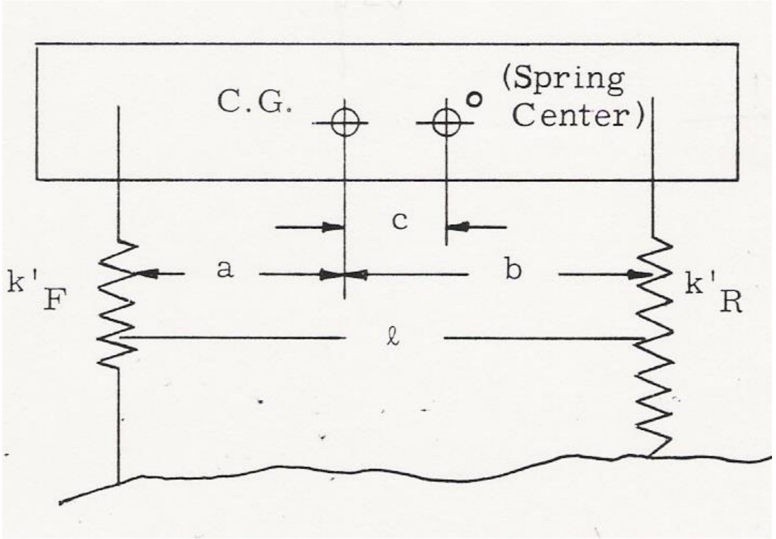
$$\gamma = \left(\frac{a^2k'_F + b^2k'_R}{J}\right)$$

$$\Rightarrow$$

$$\ddot{z} + \alpha z + \beta\theta = 0$$ Bounce

$$\ddot{\theta} + \gamma\theta + \left(\frac{M\beta}{J}\right)z = 0$$ Pitch

Two Degree of Freedom Undamped Primary Ride Model



$$\alpha = \left(\frac{k'_F + k'_R}{M}\right)$$

$$\beta = \left(\frac{bk'_R - ak'_F}{M}\right)$$

$$\gamma = \left(\frac{a^2 k'_F + b^2 k'_R}{J}\right)$$

$$\ddot{z} + \alpha z + \beta\theta = 0$$

$$\ddot{\theta} + \gamma\theta + \left(\frac{M\beta}{J}\right) z = 0$$

(1)

$$z = A_z \cos\omega t$$

$$\theta = A_z \cos\omega t$$

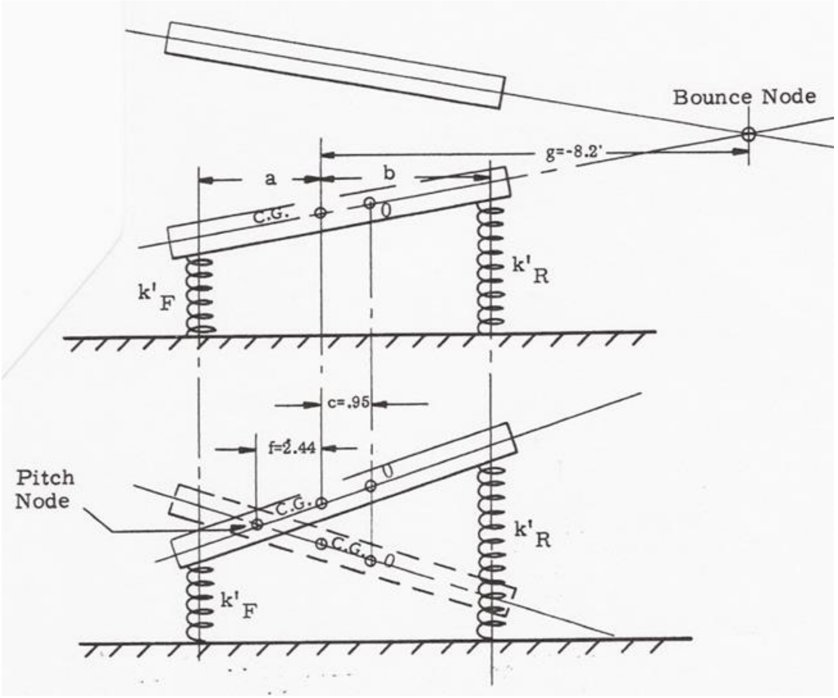
$$\frac{A_z}{A_\theta} = \frac{\beta}{\omega^2 + \alpha}$$

$$\frac{A_z}{A_\theta} = -\left(\frac{\gamma - \omega^2}{\beta}\right)\left(\frac{J}{M}\right)$$

$$\omega_{1,2}^2 = \left(\frac{\alpha + \gamma}{2}\right) \pm \sqrt{\left(\frac{\alpha - \gamma}{2}\right)^2 + \frac{M\beta^2}{J}}$$

(1) Recall that the solution to a 2nd order differential equation takes the form of A*cos(ωt)

Two Degree of Freedom Undamped Primary Ride Model



$$\alpha = \left(\frac{k_F' + k_R'}{M}\right)$$

$$\beta = \left(\frac{bk_R' - ak_F'}{M}\right)$$

$$\gamma = \left(\frac{a^2 k_F' + b^2 k_R'}{J}\right)$$

$$\frac{A_z}{A_\theta} = \frac{\beta}{\omega^2 + \alpha}$$

$$\frac{A_z}{A_\theta} = -\left(\frac{\gamma - \omega^2}{\beta}\right)\left(\frac{J}{M}\right)$$

Pitch and Bounce Amplitude Ratio's (aka Nodes)

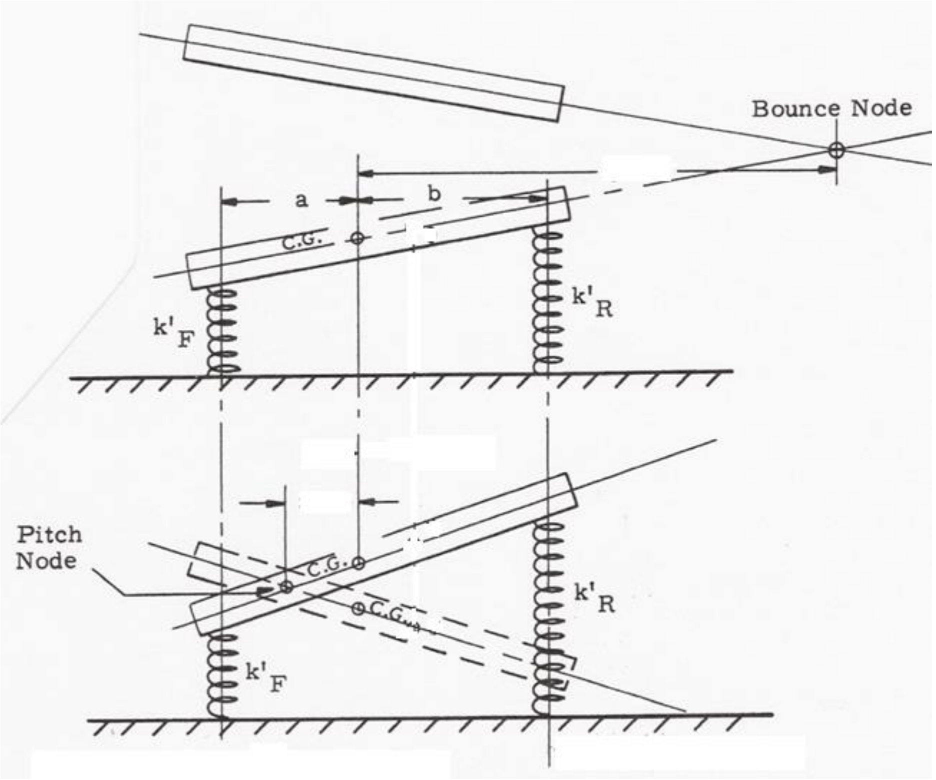
Units = m/rad

Pitch and Bounce Natural Frequencies
Units = rad/s

$$\omega_{1,2}^2 = \left(\frac{\alpha + \gamma}{2}\right) \pm \sqrt{\left(\frac{\alpha - \gamma}{2}\right)^2 + \frac{M\beta^2}{J}}$$

(1) Recall that the solution to a 2nd order differential equation takes the form of A*cos(ωt)

Two Degree of Freedom Undamped Primary Ride Model



Q: Which pair of amplitude ratio and natural frequency is pitch and which is bounce?

A: We will show with an example

$$\frac{A_z}{A_\theta} = \frac{\beta}{\omega^2 + \alpha}$$

$$\frac{A_z}{A_\theta} = -\left(\frac{\gamma - \omega^2}{\beta}\right)\left(\frac{J}{M}\right)$$

Pitch and Bounce Amplitude Ratio's (aka Nodes)

Units = m/rad

Pitch and Bounce Natural Frequencies
Units = rad/s

$$\omega_{1,2}^2 = \left(\frac{\alpha + \gamma}{2}\right) \pm \sqrt{\left(\frac{\alpha - \gamma}{2}\right)^2 + \frac{M\beta^2}{J}}$$

(1) Recall that the solution to a 2nd order differential equation takes the form of A*cos(ωt)

Example – Simplified 2-DOF Undamped Primary Ride Model

(green)	input
(yellow)	calculated

2019 Jeep Cherokee Trailhawk AWD @ SSF loading condition

Inputs

a	mm	1169.6	cg to frt axle
b	mm	1552.2	cg to rear axle
M_V	kg	2083.0	mass
$M_{V,F}$	kg	1187.9	front mass
$M_{V,R}$	kg	895.1	rear mass
m_F	kg	118.8	front unsprung mass (assume 10% of total front mass)
m_R	kg	89.5	rear unsprung mass (assume 10% of total rear mass)
M_F	kg	1069.1	front sprung mass (assume 10% unsprung)
M_R	kg	805.6	rear sprung mass (assume 10% unsprung)
M	kg	1874.7	Sprung mass
K_F	N/mm	73.9	2x front suspension rate
K_R	N/mm	81.0	2x rear suspension rate
$K_{T,F}$	N/mm	600.0	2x tire rate
$K_{T,R}$	N/mm	600.0	2x tire rate
K'_F	N/mm	65.8	LF + RF ride rate
K'_R	N/mm	71.3	LR + RR ride rate
l	kgm^2	2721.8	wheelbase
J_V	kgm^2	3526.0	Total vehicle inertia
J	kgm^2	3147.8	Sprung mass pitch inertia

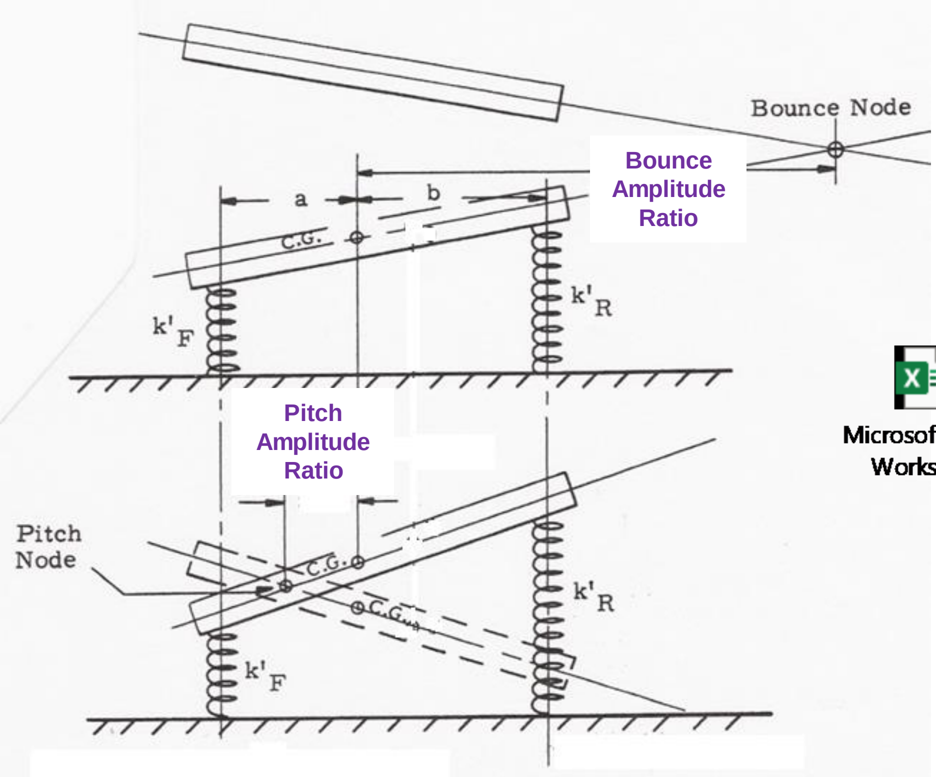
Outputs

Pitch/Bounce Natural Frequencies

ω_1	9.64	rad/s	1.53	hz
ω_2	7.96	rad/s	1.27	hz

Pitch/Bounce Amplitude Ratios

$A_{Z1}/A_{\theta 1}$	0.91	m/rad	909.44	mm/rad
$A_{Z2}/A_{\theta 2}$	-1.85	m/rad	-1846.31	mm/rad



Microsoft Excel Worksheet

$$\left(\omega_1, \frac{A_{z1}}{A_{\theta 1}}\right) = (1.53, 909.44)$$

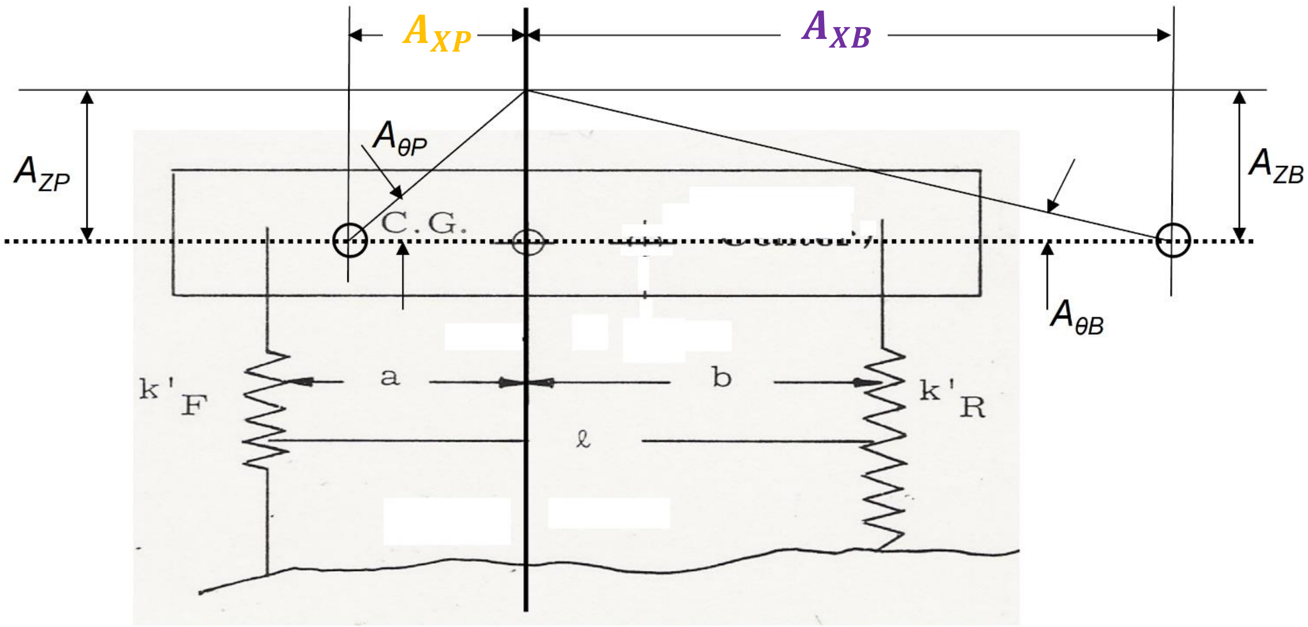
$$\left(\omega_2, \frac{A_{z2}}{A_{\theta 2}}\right) = (1{,}27, -1849.31)$$

Ok, but which pair is pitch and which is bounce?

The 2-DOF Primary Ride Model

Q: How do we relate ‘amplitude ratio’ to ‘node location’

A: By the circular arc equation; s = rθ



Utilizing small angle approximation:

Pitch:

$$A_{ZP} \approx A_{XP}A_{\theta P}$$

$$A_{XP} \approx \frac{A_{ZP}}{A_{\theta P}}$$

Bounce:

$$A_{ZB} \approx A_{XB}A_{\theta B}$$

$$A_{XB} \approx \frac{A_{ZP}}{A_{\theta B}}$$

Example – Simplified 2-DOF Undamped Primary Ride Model

input
calculated
2019 Jeep Cherokee Trailhawk AWD @ SSF loading condition

Inputs

a	mm	1169.6	cg to frt axle
b	mm	1552.2	cg to rear axle
M_V	kg	2083.0	mass
$M_{V,F}$	kg	1187.9	front mass
$M_{V,R}$	kg	895.1	rear mass
m_F	kg	118.8	front unsprung mass (assume 10% of total front mass)
m_R	kg	89.5	rear unsprung mass (assume 10% of total rear mass)
M_F	kg	1069.1	front sprung mass (assume 10% unsprung)
M_R	kg	805.6	rear sprung mass (assume 10% unsprung)
M	kg	1874.7	Sprung mass
K_F	N/mm	73.9	2x front suspension rate
K_R	N/mm	81.0	2x rear suspension rate
$K_{T,F}$	N/mm	600.0	2x tire rate
$K_{T,R}$	N/mm	600.0	2x tire rate
K'_F	N/mm	65.8	LF + RF ride rate
K'_R	N/mm	71.3	LR + RR ride rate
l	kgm^2	2721.8	wheelbase
J_V	kgm^2	3526.0	Total vehicle inertia
J	kgm^2	3147.8	Sprung mass pitch inertia

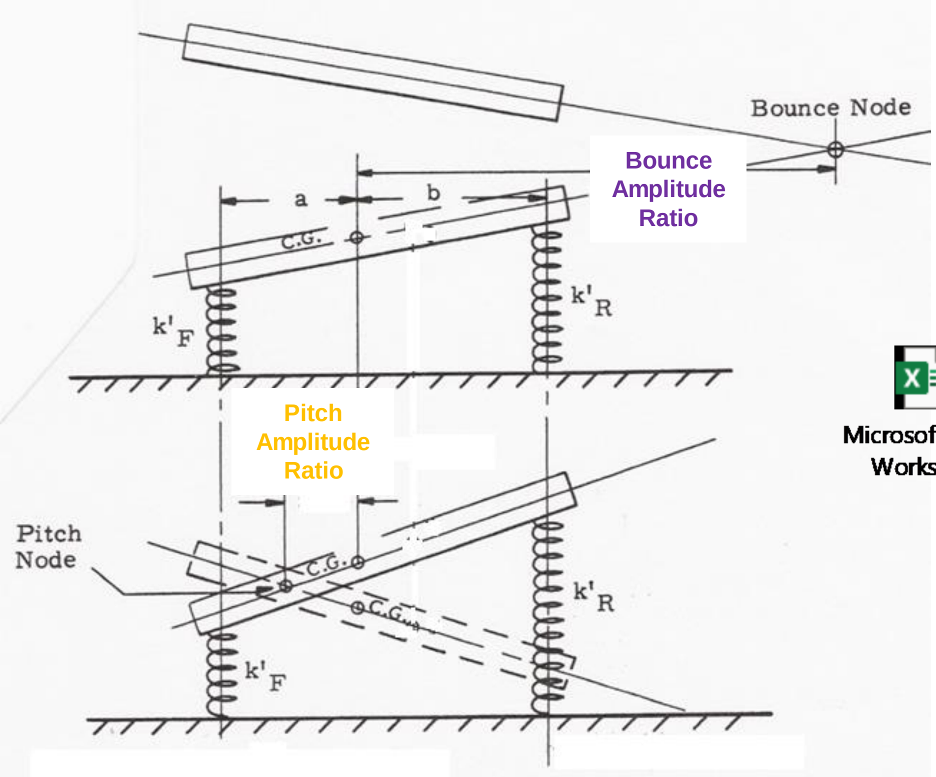
Outputs

Pitch/Bounce Natural Frequencies

ω_1	9.64	rad/s	1.53	hz
ω_2	7.96	rad/s	1.27	hz

Pitch/Bounce Amplitude Ratios

$A_{Z1}/A_{\theta 1}$	0.91	m/rad	909.44	mm/rad
$A_{Z2}/A_{\theta 2}$	-1.85	m/rad	-1846.31	mm/rad



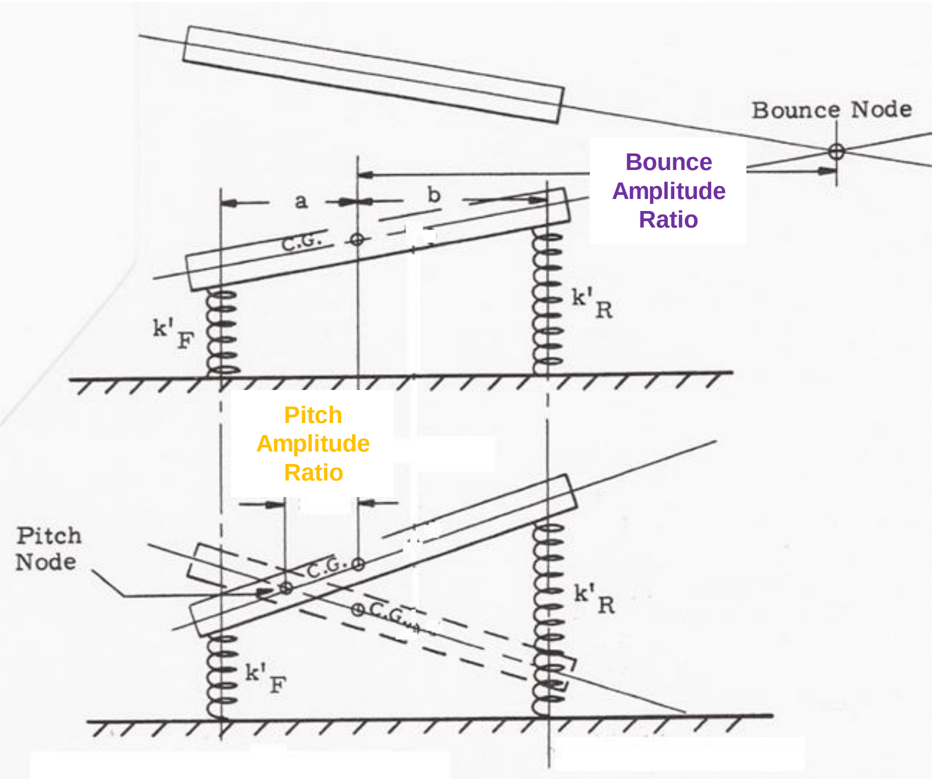
Microsoft Excel Worksheet

$$\left(\omega_1, \frac{A_{z1}}{A_{\theta 1}}\right) = (1.53, 909.44)$$ **Pitch**

$$\left(\omega_2, \frac{A_{z2}}{A_{\theta 2}}\right) = (1{,}27, -1849.31)$$ **Bounce**

Some comments about the simplified 2-DOF ride model

- The pitch and bounce nodes are centers of rotational oscillation
- The **pitch** node will always be the one inside the wheelbase
- The **bounce** node will always be the one outside the wheelbase
- When the sprung mass is excited at the **pitch** natural frequency, the driver will feel mainly feel a **pitching** sensation since the node is close to the driver
- When the sprung mass is excited at the **bounce** natural frequency, the driver will feel mainly a **bounce** sensation, even though the sprung mass is rotating around a node, albeit far away from the driver



A common 'rule of thumb' is to have the pitch node close to the drivers h-point and the bounce node as far away from sprung mass Cg as possible.

Maurice Olley and Guidelines for Primary Ride

Maurice Olley was a mechanical engineer with Rolls Royce and General Motors, whose career stretched from the late 1920's until the early 60's. In the **1930's**, while Olley was a suspension engineer with Cadillac, he conducted extensive testing relating to ride balance and comfort.

According to Olley, there exists a relationship between front and rear wheel rates that provides desirable primary pitch and bounce characteristics in the absence of shock damping. Once this relationship is established, the development engineer needs to apply minimal shock damping to finely tune the primary ride.

This follows the general development principal that the best shock control for ride is the least shock control that will do the job.

In Part 2 of this discussion - Primary Ride Tuning Guidelines - we will examine Olley's Criteria for good primary ride in the absence of damping

Summary

- Primary Ride pertains to the low frequency, relatively high amplitude rigid body motion of the vehicle sprung mass
- Vertical road disturbance displacement and velocity can be calculated as a function of vehicle speed, road amplitude and road wavelength
- Sprung mass, unsprung mass, suspension stiffness and damping and tire stiffness and damping are inputs to the 4-dof ride model
- Examination of the 4-dof model equations reveal estimates for wheel hop and body-on-chassis sprung mass natural frequencies

Summary

- Sprung mass and inertia and ride stiffness and damping are inputs to the the 2-dof primary ride model

- Closed form solution of the 2-dof model equations reveal estimates for the sprung mass pitch and bounce natural frequencies and corresponding amplitude ratios (nodes)

- Results can be compared to historical design guidelines (‘rule of thumb’) for good primary ride in the absence of damping
  - More to come in Part 2 - Primary Ride Tuning Guidelines

Parameter Definitions

A_Z(L) = bounce amplitude
A_θ(L) = pitch amplitude
a(L) = horizontal distance between the front axle and the sprung mass center of gravity
$\alpha(t^{-2})$ = equation coefficient
b(L) = horizontal distance between the rear axle and the sprung mass center of weight
$\beta(Lt^{-2})$ = equation coefficient
C.G. = sprung mass center of gravity
$c_F(FtL^{-1})$ = front damping coefficient (double the vehicle corner damping coefficient)
$c_R(FtL^{-1})$ = rear damping coefficient (double the vehicle corner damping coefficient)
$c_T(FtL^{-1})$ = tire vertical damping coefficient
$\gamma(t^{-2})$ = equation coefficient
f_B(Hz) = body-on-chassis bounce natural frequency
f_{HF}(Hz) = wheel hop natural frequency
f_P(Hz) = body-on-chassis pitch natural frequency
$J(ML^2)$ = sprung mass pitch moment of inertia relative to the sprung mass center of gravity
$k_F(FL^{-1})$ = front suspension rate (double the vehicle corner suspension rate)
$k'_F(FL^{-1})$ = front suspension ride rate (double the vehicle rate)
$k_R(FL^{-1})$ = rear suspension rate (double the vehicle corner suspension rate)
$k'_R(FL^{-1})$ = rear suspension ride rate (double the vehicle corner rate
$k_T(FL^{-1})$ = tire vertical static spring rate
l(L) = wheel base
M(M) = sprung mass (also called body mass)
m_F(M) = front axle unsprung mass
m_R(M) = rear axle unsprung mass
y_F(L) = vertical displacement of the road at the front axle
y_R(L) = vertical displacement of the road at the rear axle
z(L) = vertical displacement of the sprung mass C.G. relative to its static position
z_F(L) = vertical displacement of the front unsprung mass C.G. relative to its static position
z_R(L) = vertical displacement of the rear unsprung mass C.G. relative to its static position
θ(rad) = sprung mass pitch angle relative to horizontal
ω_B(rad/s) = body bounce natural frequency
ω_{HF}(rad/s) = wheel hop frequency
ω_P(rad/s) = body pitch natural frequency