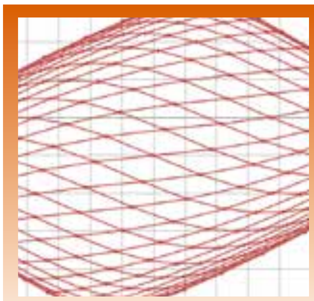


Slip Angle provides a summary of OptimumG's seminars

Getting more from your yaw diagrams

Our analysis of yaw versus lateral acceleration continues with Claude Rouelle's explanation of the yaw moment diagram and how to interpret it

We will start this article by reviewing some basic concepts. As we have seen in the previous articles on the yaw moment versus lateral acceleration method, an understeering car is defined as a car that doesn't have enough yaw moment and an oversteering car is a car with too much yaw moment. The yaw moment is caused by 12 factors, which come from the tyres: the four lateral forces (F_y), the four longitudinal forces (F_x) and the four self-aligning torques (M_z). Let's also remind ourselves that at the corner apex – i.e. where the car is closest to a steady-state – the resultant yaw moment should be equal to zero.

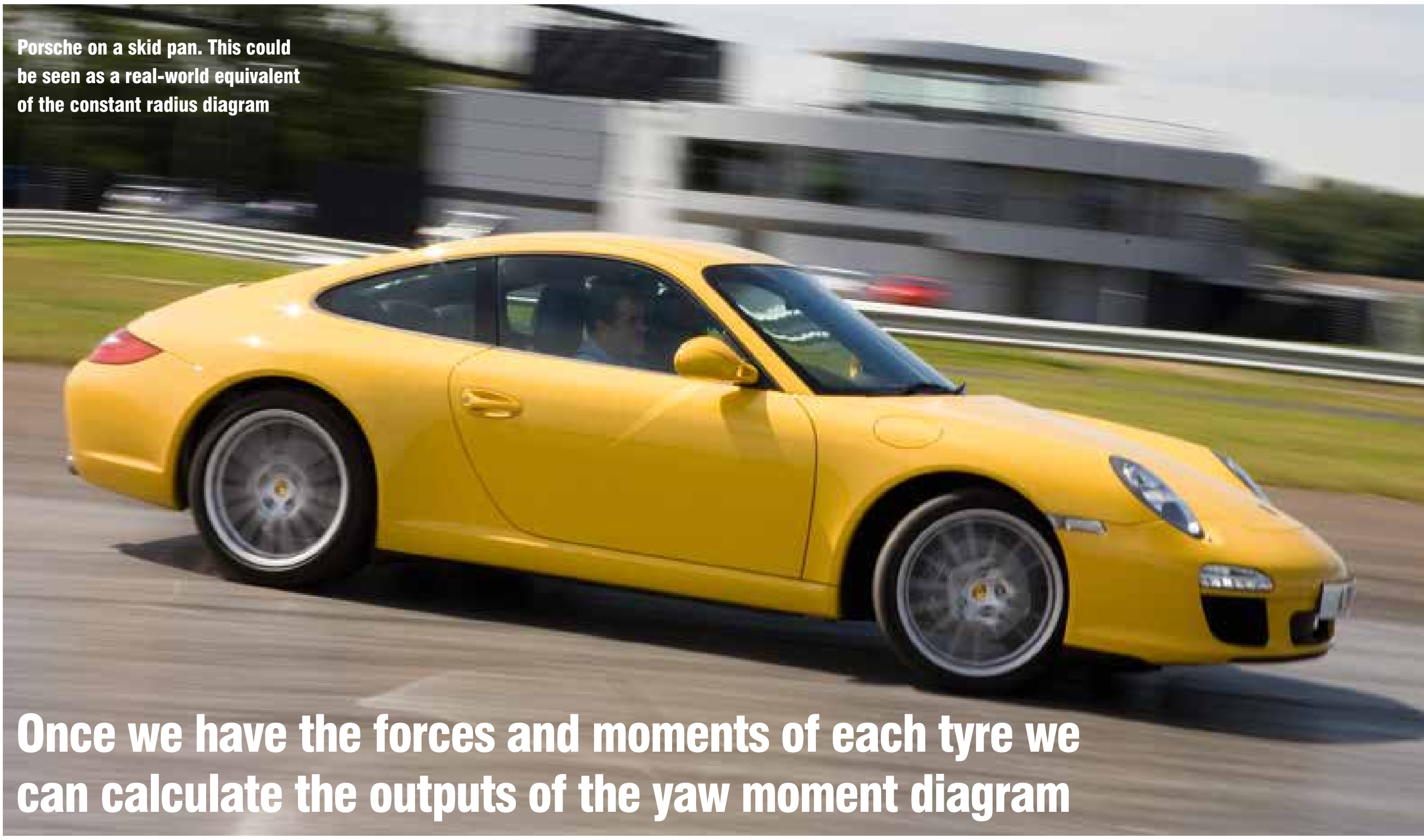
The yaw moment diagram is a 2D chart that represents two outputs: the resultant yaw moment in the vertical axis and the resultant lateral acceleration in the horizontal axis. Each point of this diagram represents a combination of two inputs: the steering angle δ and the CG slip angle (also called CG yaw angle) ß. Using the yaw moment diagram, an engineer can get a car design and set-up that can not only reach the maximum possible lateral acceleration, but also have the right amount of yaw moment at the right time during a corner.

There are two types of yaw moment diagrams: at constant speed and at constant radius **(Figure 1).** In both cases, the inputs (steering angle δ and CG slip angle) and outputs (lateral acceleration and yaw moment) are the same, but they represent different scenarios. The constant speed simulation represents the car running at a very large flat surface, while the driver tries to reach the tightest possible radius in a steady-state, neutral behaviour, at a given speed. The constant radius method represents, of course, the car at a skid pad, where the driver tries to reach the maximum longitudinal speed also in steady-state.

Let's now dive into the algorithm that generates the yaw moment diagram. As said previously, the two main inputs of the diagram are the steering angle δ and the CG slip angle ß. The procedure described in the next paragraphs must be repeated for every combination of these two parameters, within a range, to generate the full diagram. Since it is not possible to calculate the equilibrium state of the vehicle explicitly, we will need to run an iterative process until a convergence criterion is met.

Steering angle

We start by positioning the vehicle with a given steering angle δ and CG slip angle ß **(Figure 2)** at either a constant speed or radius. Then, we can calculate the slip angle of each



Porsche on a skid pan. This could be seen as a real-world equivalent of the constant radius diagram

Once we have the forces and moments of each tyre we can calculate the outputs of the yaw moment diagram

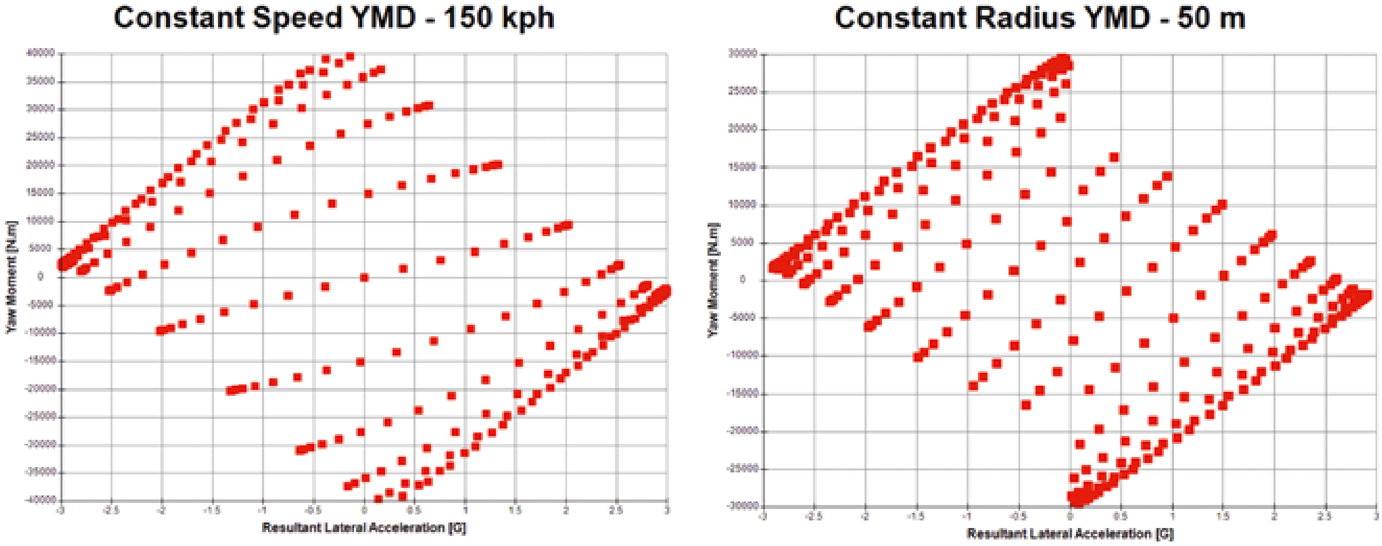


Figure 1: The two types of yaw moment versus lateral acceleration diagrams – constant speed or constant radius

tyre, providing we have some basic information about the car, such as the steering ratio, the Ackermann geometry and the toe angles of each wheel. The formulae for calculating slip angles are shown in **Figure 3**. Note that, at this stage, we assume that the vehicle has zero yaw velocity (r), which will be included in the next iterations. Basically, there are three causes for tyre slip angle: steering input (or toe angle) δ, CG slip angle ß and yaw velocity r. With these three pieces of information, we can calculate the longitudinal and lateral speeds at each contact patch and obtain the slip angle.

Each tyre slip angle comes this way **(Figure 3)**:

$\alpha LF = [V_y + (r\ a)] / [V_x - (r\ T_f/2)] - \delta_{LF}$ (9)

$\alpha RF = [V_y + (r\ a)] / [V_x + (r\ T_f/2)] - \delta_{RF}$ (10)

$\alpha LR = [V_y - (r\ b)] / [V_x - (r\ T_r/2)]$ (11)

$\alpha RR = [V_y - (r\ b)] / [V_x + (r\ T_r/2)]$ (12)

This is where V_x and V_y are the longitudinal and lateral car CG speed, T_f and T_r are the front and rear tracks, *a* is the distance from the front axle to the CG, *b* is the distance from the CG to the rear axle, ß is the CG slip angle, *r* is the yaw velocity and δ_{LF} and δ_{LR} are the left front and right front steering angles. In this case the bump steer and the compliance are not being considered.

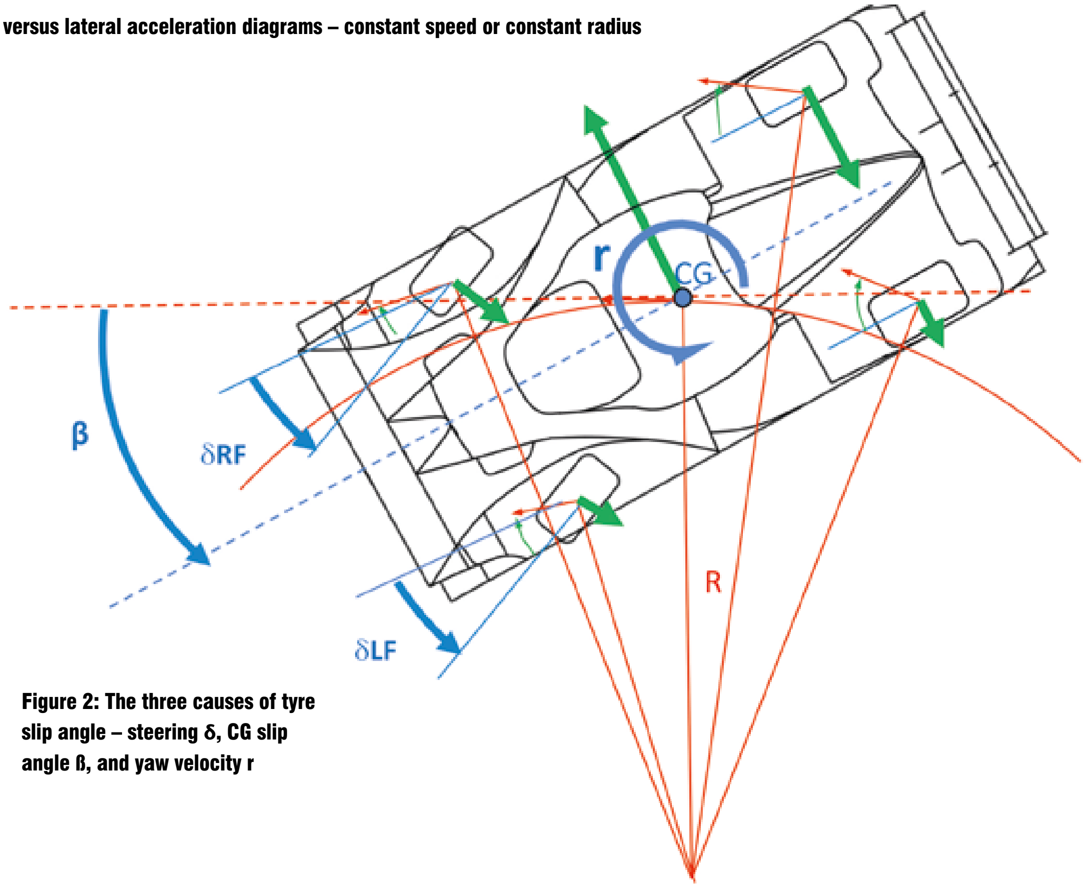


Figure 2: The three causes of tyre slip angle – steering δ, CG slip angle ß, and yaw velocity r

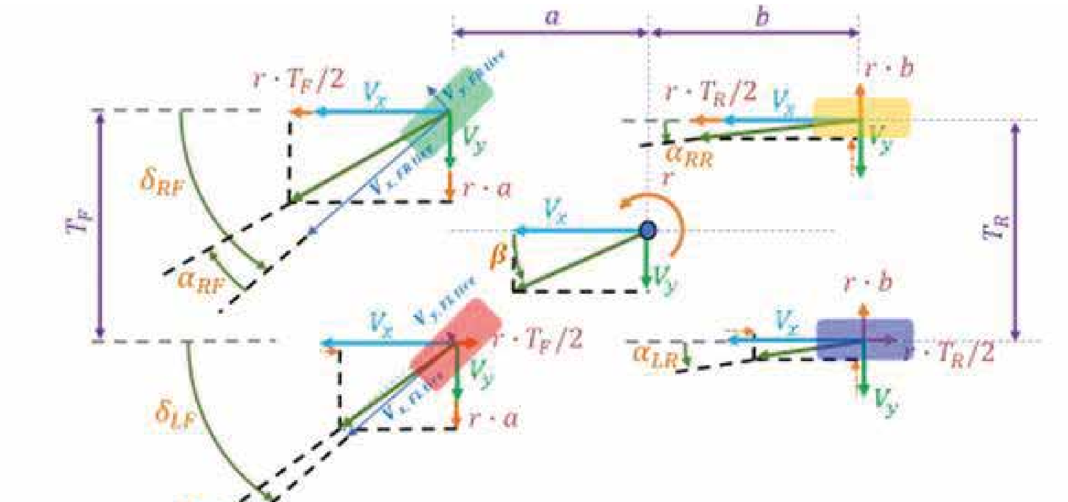
Black magic

Once we have the slip angles we can proceed to calculate the forces and moments of each tyre. To do so, we can use a simple 'magic formula' tyre model. The magic formula tyre model will receive as inputs the tyre slip angle α, the vertical force Fz, the camber angle γ and the slip ratio κ (which, in this case, is zero). Some tyre models can also include effects of speed, pressure and temperature, but let's keep it simple for now. The tyre model will then return the values of lateral force (F_y), longitudinal force (F_x) and self-aligning moment (M_z).

Now that we have the forces and moments of each tyre, we can calculate the outputs of the yaw moment diagram: the lateral acceleration and the yaw moment. However, since the resultant lateral acceleration is (most likely) different than zero, the values of vertical forces that we initially used in the tyre model for calculating the forces and moments are no longer correct. The lateral acceleration will cause a lateral load transfer, which will modify the vertical loads on each tyre. The inclination angles will also be different, as the lateral acceleration results in a roll angle. Even the resultant slip angles will change, since now we have a yaw velocity r = Ay/Vx. If we are running a constant radius simulation, the new value of longitudinal speed must also be calculated (V=√(A R)). Thus, we need to update all these values and recalculate the forces and moments of the tyres. We iterate through this process until the imposed lateral acceleration matches the resultant lateral acceleration. More details on this algorithm are explained in the OptimumG seminars.

It is worth mentioning that the accuracy and usefulness of the yaw moment diagram will change depending on how complex our vehicle model is. You can, for example, consider effects such as suspension kinematics (bump steer, camber change, etc.), compliance, changes in aerodynamic forces with ride heights, chassis stiffness,

The lateral acceleration will cause a lateral load transfer, which will modify the vertical loads on each tyre



$$V_y = V_x \cdot \beta$$

$$\alpha_{RF} = \frac{V_y + r \cdot a}{V_x + r \cdot T_F/2} - \delta_{RF} \qquad \alpha_{RR} = \frac{V_y - r \cdot b}{V_x + r \cdot T_R/2}$$

$$\alpha_{LF} = \frac{V_y + r \cdot a}{V_x - r \cdot T_F/2} - \delta_{LF} \qquad \alpha_{LR} = \frac{V_y - r \cdot b}{V_x - r \cdot T_R/2}$$

Figure 3: Tyre slip angle calculation

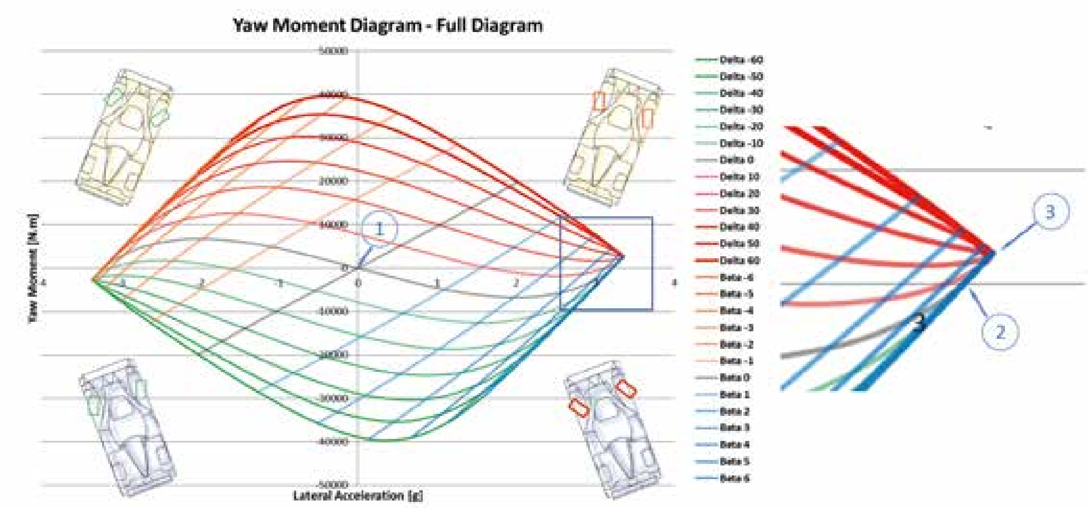


Figure 4: Yaw moment diagram at constant speed with CG slip angle and steering isolines

longitudinal acceleration, etc. As a general rule, the more complex the vehicle model is, the more sensitivity analysis we can perform.

Once we calculate the vehicle state for every combination of steering angle δ and CG slip angle ß, we will have the full yaw moment diagram. Ideally, some measurements made on track can help us to decide the realistic window of inputs. To better visualise the diagram, we usually connect lines with same CG slip angle or same steering angle, as shown in **Figure 4**.

Point taken

Even though we calculate the full yaw moment diagram, we are usually only interested in a few of its points. Obviously, we don't know which points will fall in the regions of interest prior to the simulation, that's why we compute all of them. **Figure 4** shows three points that we often watch for, when interpreting a yaw diagram of constant speed.

At point 1, we see that the intersection of the isoline $\delta = 0$ and the isoline ß = 0. This point usually corresponds to the origin of the outputs, where both lateral acceleration and yaw moment are equal to zero. That seems logical: at constant speed with no yaw angle and no steering input the car should be travelling in a straight line; the turn radius is infinite. There is an exception: if the car is asymmetrical (in NASCAR or Indycar, on ovals, for example) it is possible that, even without any steering or CG slip angle input, the car would have a small amount of yaw moment and lateral acceleration. This point is particularly important because we can take conclusions about the behaviour of the car as it enters the corner. This subject will be further explored in the next article.

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